Mathematics IV, Exercises 5.

Corrections May 28.

1). Entangled states (Verschränkter Zustand).

Consider the Hilbert $\mathcal{H}^{(1/2)}$ space on the field $\mathbb{C}$ and spanned by the vectors $\mathbf{u} = (\mathbf{1}, \mathbf{0})$ and $\mathbf{d} = (\mathbf{0}, \mathbf{1})$ and with scalar product satisfying $\langle u|u\rangle = \langle d|d\rangle = 1$ and $\langle d|u\rangle = 0$. Show that the vector $f \in \mathcal{H}^{(1/2)} \otimes \mathcal{H}^{(1/2)}$, $f = 1/\sqrt{2}\,(\mathbf{u} \otimes \mathbf{d} - \mathbf{d} \otimes \mathbf{u})$ can not be written as $f_1 \otimes f_2$ with $f_1 \in \mathcal{H}^{(1/2)}$ and $f_2 \in \mathcal{H}^{(1/2)}$.

2). The Exponential function.

The exponential function of a complex variable is defined by the Taylor series:

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

a). Show that the convergence radius, R, is infinite. Use the definition to show that

$$e^{z_1} e^{z_2} = e^{z_1 + z_2}$$
$$e^z = e^x\,(\cos(y) + i\sin(y)) \quad \text{with} \quad z = x + iy$$

b) Let $z = e^{2\pi i/n}$, $n > 1$, $n \in \mathbb{N}$. Show that

$$\sum_{k=0}^{n-1} z^k = 0$$

3).

Show that:

$$\cos^4(\phi) = 3/8 + \cos(2\phi)/2 + \cos(4\phi)/8$$
$$\arcsin(z) = -i \ln\left(iz + \sqrt{1 - z^2}\right)$$