Mathematics IV, Exercises 11.

Corrections July 9.

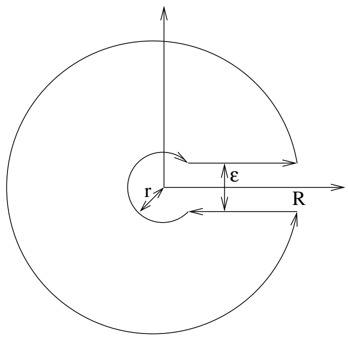
Integrals and sums

1)

Compute the following integrals:

1. $$\int_0^{\pi/2} \frac{dx}{1+\sin^2(x)}$$
2. $$\int_{-\infty}^{\infty} \frac{x^2 dx}{x^4+6x^2+13}$$
3. $$\int_0^{\infty} \frac{\sqrt{x}dx}{16+x^2}$$

For the integral 3. Do not forget that the square root has branching points at $z=0$ and $z=\infty$ so that the branching line can be defined as the positive real axis. As a starting point, you can choose the path shown in the figure. After taking the limit $\epsilon \to 0$ (beware, in doing so you will be crossing a branching line!) you can take the limits $r \to 0$ and $R \to \infty$.



2 Matsubara sums.

Consider the Matsubara frequencies: $\omega_m = (2m+1)\pi/\beta$, $m \in \mathbb{Z}$ and $\beta > 0$. Compute:

$$\frac{1}{\beta}\sum_{m=-\infty}^{\infty} \frac{e^{i\tau\omega_m}}{E - i\omega_m}$$

both for $\beta > \tau > 0$ and $-\beta < \tau < 0$. Here is a hint. For the case $\beta > \tau > 0$, consider the function $\frac{e^{\tau z}}{e^{\beta z}+1}\frac{1}{E-z}$ and carry out the integral over a circle of radius R. Then take the limit $R \to \infty$.