QM1 Exercises. Sheet 5.

Corrections June 12-13

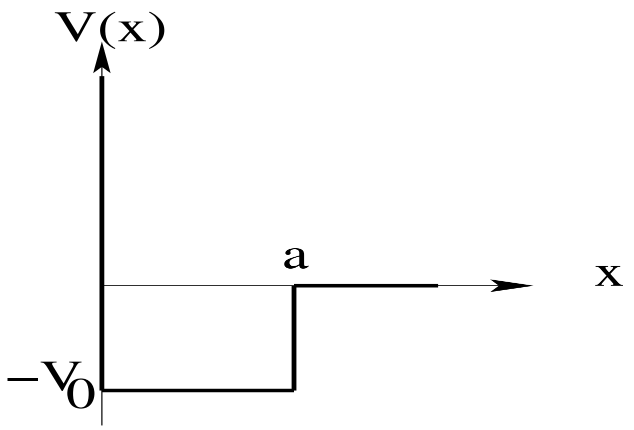
1) **Revival of quantum states.** (5 Points)

Let E_n be the energy spectrum of a quantum mechanical system described by the Hamiltonian $\hat{H}$. That is; $\hat{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$. Assume the the energy levels are equidistant as in the Harmonic oscillator for example. Show that for any initial state $\Psi(x, t = 0)$ there is a revival time t_{revival} such that $\Psi(x, t_{\text{revival}}) = e^{i\phi}\Psi(x, t = 0)$.

Such phenomena has been beautifully demonstrated in the context of Bose Einstein Condensates. Here is the reference: Nature 419, 51-54 (5 September 2002), Collapse and revival of the matter wave field of a Bose Einstein condensate by Markus Greiner, Olaf Mandel, Theodor W. Hänsch and Immanuel Bloch.

2) Potential barrier. (10 Points)

Consider the following potential in one dimension.



$$V(x) = \begin{cases} \infty & \text{for } x < 0 \\ -V_0 & \text{for } 0 < x < a \\ 0 & \text{for } x > a \end{cases} \tag{1}$$

a) Find the stationary states for $E < 0$. For those states the wave function falls off exponentially at $x \to \infty$. They are hence normalizable and are called bound states.

b) Find the stationary states for $E > 0$. Theses states extend to infinity and are hence not normalizable. They are referred to as scattering states.

c) For $E > 0$ find the phase relation between the incident and reflected waves.

Note. In class we have seen that when $V(x)$, and E are bounded then the wave function and it's first derivative are continuous. In the present situation the potential diverges for $x < 0$ and at $x = 0$ the wave function is continuous but not differentiable !