**QM1 Exercises. Sheet 9.**

**Corrections July 10-11**

**1) Central Potentials** (5 Points)

A particle of mass $m$ is confined to the interior of a spherical hollow cavity of radius $R$ with impenetrable walls.

**a)** Compute the ground state wave function and energy.

**b)** Find an expression for the pressure exerted on the walls of the cavity by the particle in its ground state.

**2) Hydrogen atom** (5 Points)

At time $t = 0$ the wave function of a hydrogen atom is

$$\Psi(\boldsymbol{x}, t = 0) = \frac{1}{\sqrt{10}} \left(2\Psi_{100}(\boldsymbol{x}) + \Psi_{210}(\boldsymbol{x}) + \sqrt{2}\Psi_{211}(\boldsymbol{x}) + \sqrt{3}\Psi_{21-1}(\boldsymbol{x})\right) \tag{1}$$

Here the subscripts denote the quantum numbers $n$, $l$ and $m$, and we will ignore spin degrees of freedom.

**a)** Compute the average energy.

**b)** Compute the time evolution of the wave function.

**c)** Given an apparatus which measures $\hat{L}^2$ and $\hat{L}_z$ what is the probability of obtaining the result $l = 1$ and $m = 1$ at time $t$. What is the form of the wave function directly after the measurement.

**d)** If we now measure $\hat{L}^2$ and $\hat{L}_x$ what is the probability of obtaining the result $l = 1$ and $m_x = 1$ at time $t$. Note that $m_x$ is defined as $\hat{L}_x|l, m_x\rangle_x = \hbar m_x|l, m_x\rangle_x$ where we have chosen the quantization axis to be $\boldsymbol{e}_x$ rather that $\boldsymbol{e}_z$. Again, what is the form of the wave function directly after the measurement.

**e)** What is the probability at time $t = 0$ of finding the electron within a distance $10^{-10}$ cm of the proton.

**3) Spin 1 systems.** (5 Points)

Consider a particle of spin 1, and whose spin dynamics is described by the Hamiltonian:

$$\hat{H} = A\hat{S}_z + B\hat{S}_x^2. \tag{2}$$

Here, $A$ and $B$ are constants.

**a)** Calculate the energy levels of the system.

**b)** If at time $t = 0$ the system is in a state $|\Psi\rangle$ with $\hat{S}_z|\Psi\rangle = \hbar|\Psi\rangle$ compute the expectation value of the spin operator $\hat{\boldsymbol{S}}$ at time $t$.