Metals, insulators, topological insulators.

Summary sheets.

Current operator.

Classically.

$$\mathbf{j} = e\mathbf{v} \qquad L = \frac{m\mathbf{v}^2}{2} - e(\phi - \mathbf{v}\mathbf{A}/c)$$

$$\mathbf{p} = \frac{\partial}{\partial \mathbf{v}} L \quad = m\mathbf{v} + \frac{e}{c}\mathbf{A}$$

$$\mathbf{j} = \frac{e}{m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)$$

QM

$$\hat{\mathbf{j}} = \sum_{s.s'} \int d\mathbf{x}^3 d\mathbf{x}'^3 \quad \hat{\psi}_s^\dagger(\mathbf{x}) \left\langle \mathbf{x}, s \right| \frac{e}{m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\hat{\mathbf{x}}, t)\right) \left| \mathbf{x}', s' \right\rangle \hat{\psi}_{s'}^\dagger(\mathbf{x}')$$

Note:

$$\left\langle \mathbf{x} \right| \hat{\mathbf{P}} \left| \mathbf{x}' \right\rangle \quad = \frac{\hbar}{i} \nabla_{\mathbf{x}} \delta^3(\mathbf{x} - \mathbf{x}')$$

$$\hat{\mathbf{j}} = \int d\mathbf{x}^3 \quad \hat{\mathbf{j}}^P(\mathbf{x}) + \hat{\mathbf{j}}^D(\mathbf{x}),$$

$$\hat{\mathbf{j}}^P(\mathbf{x}) = \frac{e\hbar}{2mi} \sum_s \quad \hat{\psi}_s^\dagger(\mathbf{x}) \nabla \hat{\psi}_s(\mathbf{x}) \quad - \left(\nabla \hat{\psi}_s^\dagger(\mathbf{x})\right) \hat{\psi}_s(\mathbf{x})$$

$$\hat{\mathbf{j}}^D(\mathbf{x}) = -\frac{e^2}{mc} \sum_s \quad \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_s(\mathbf{x})\, \mathbf{A}(\mathbf{x}, t)$$

Hamilton Operator.

$$\hat{H} = \sum_{s.s'} \int d\mathbf{x}^3 d\mathbf{x}'^3 \quad \hat{\psi}_s^\dagger(\mathbf{x}) \left\langle \mathbf{x}, s \right| \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\hat{\mathbf{x}}, t)\right)^2 + V_{ion}(\hat{\mathbf{x}}) \left| \mathbf{x}', s' \right\rangle \hat{\psi}_{s'}^\dagger(\mathbf{x}') =$$

$$\sum_s \int d\mathbf{x}^3 \quad \hat{\psi}_s^\dagger(\mathbf{x}) \left\{ \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}(\mathbf{x}, t)\right)^2 + V_{ion}(\mathbf{x}) \right\} \hat{\psi}_s^\dagger(\mathbf{x})$$

$$\hat{\mathbf{j}}(\mathbf{x}) = \quad \hat{\mathbf{j}}^P(\mathbf{x}) + \hat{\mathbf{j}}^D(\mathbf{x}) = -c \, \frac{\delta \hat{H}(\mathbf{A})}{\delta \mathbf{A}(\mathbf{x})}$$

1) Linear response to electric field.

Peierls Phase factors:

$$\hat{H}(\mathbf{A}) = \sum_{\mathbf{i},\mathbf{j},m.n} c^{\dagger}_{\mathbf{i},n}\ t_{n,m}(\mathbf{i}-\mathbf{j})c_{\mathbf{j},m} \exp\left[\frac{2\pi i}{\Phi_0}\int_{\mathbf{i}}^{\mathbf{j}} \mathbf{A}(\mathbf{l},t)\bullet d\,\mathbf{l}\right] + \quad \hat{H}_{el-el}$$

Current Operator:

$$\hat{\mathbf{j}}_{\alpha}(\mathbf{r}) = -c\ \frac{\delta \hat{H}(\mathbf{A})}{\delta \mathbf{A}_{\alpha}(\mathbf{r})} \ =$$

$$-c\frac{2\pi i}{\Phi_0}\sum_{\boldsymbol{\delta},n,m} \boldsymbol{\delta}\bullet\mathbf{e}_{\alpha}\ \left[c^{\dagger}_{\mathbf{r},n}t_{n,m}(-\boldsymbol{\delta})c_{\mathbf{r}+\boldsymbol{\delta},m}\exp\left[\frac{2\pi i}{\Phi_0}\int_{\mathbf{r}}^{\mathbf{r}+\boldsymbol{\delta}}\mathbf{A}\bullet d\,\mathbf{l}\right] - c^{\dagger}_{\mathbf{r}+\boldsymbol{\delta},n}t_{n,m}(\boldsymbol{\delta})c_{\mathbf{r},m}\exp\left[\frac{2\pi i}{\Phi_0}\int_{\mathbf{r}+\boldsymbol{\delta}}^{\mathbf{r}}\mathbf{A}\bullet d\,\mathbf{l}\right]\right]$$

Vector potential.

$$\mathbf{A}(t) = \mathbf{A}(\omega)\,e^{-i(\omega+i\eta)t}, \qquad \eta = 0^{+}$$

$$\mathbf{E}(t) = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}(t) = \frac{\omega+i\eta}{c}\mathbf{A}(\omega)e^{-i(\omega+i\eta)t} \ = \ \mathbf{E}(\omega)e^{-i(\omega+i\eta)t}$$

$$\Rightarrow \mathbf{A}(\omega) = \ \frac{c}{\omega+i\eta}\mathbf{E}(\omega)$$

Linear response in **A.**

$$\hat{H}(\mathbf{A}) = \hat{H}_0 + \underbrace{\frac{2\pi}{\Phi_0}\ \mathbf{J}_p \bullet \mathbf{A}(t)}_{H_1(t)}, \qquad \hat{H}_0 = \sum_{\mathbf{i},\mathbf{j},m.n} c^{\dagger}_{\mathbf{i},n}\ t_{n,m}(\mathbf{i}-\mathbf{j})c_{\mathbf{j},m} + \hat{H}_{el-el}, \quad \hat{\mathbf{J}}_P = \sum_{\mathbf{i},\mathbf{j},m.n} i(\mathbf{j}-\mathbf{i})\, c^{\dagger}_{\mathbf{i},n}\ t_{n,m}(\mathbf{i}-\mathbf{j})c_{\mathbf{j},m}$$

$$\frac{1}{N}\sum_{\mathbf{r}}\left\langle \hat{\mathbf{j}}_{\alpha}(\mathbf{r})\right\rangle(t) = \frac{1}{N}\sum_{\mathbf{r}}\left[\left\langle \hat{\mathbf{j}}_{\alpha}(\mathbf{r})\right\rangle_0 - i\int_0^{\infty} dx \left\langle\left[\hat{\mathbf{j}}^H_{\alpha}(\mathbf{r},t),\ H^H_1(t-x)\right]\right\rangle_0\right]$$

Heisenberg representation: $H^H_1(t) = e^{itH_0} H_1(t) e^{-itH_0}$

Retain all terms linear in A (note that $\hat{\mathbf{j}}_{\alpha}(\mathbf{r})$ has an explicit dependence on A) to obtain the conductivity tensor.

$$\frac{1}{N}\sum_{\mathbf{r}}\left\langle \hat{\mathbf{j}}_{\alpha}(\mathbf{r})\right\rangle(t) = \frac{1}{N}\sum_{\mathbf{r}}\left\langle \hat{\mathbf{j}}_{\alpha}(\mathbf{r})\right\rangle(\omega)\ e^{-it(\omega+i\eta)}$$

$$\frac{1}{N}\sum_{\mathbf{r}}\left\langle \hat{\mathbf{j}}_{\alpha}(\mathbf{r})\right\rangle(\omega) = \sigma_{\alpha,\beta}(\omega)\mathbf{E}_{\beta}(\omega)$$

Optical conductivity.

$$\sigma_{\alpha,\beta}(\omega) = \frac{4\pi^2 c^2}{\Phi_0^2} \frac{1}{i(\omega + i\eta)} \frac{1}{N} \left[\left\langle K_\alpha \right\rangle_0 \delta_{\alpha,\beta} + \underbrace{i \int_0^\infty dt e^{i(\omega+i\eta)t} \left\langle \left[J_{p,\alpha}, J_{p,\beta}^H(-t) \right] \right\rangle_0}_{\equiv \Lambda_{\alpha\beta}(\omega)} \right]$$

$$\hat{H}_0 = \sum_{\mathbf{i},\mathbf{j},m.n} c_{\mathbf{i},n}^\dagger \; t_{n,m}(\mathbf{i}-\mathbf{j}) c_{\mathbf{j},m} \; + \; \hat{H}_{el-el}$$

$$\hat{\mathbf{J}}_P = \sum_{\mathbf{i},\mathbf{j},m.n} i(\mathbf{j}-\mathbf{i})\, c_{\mathbf{i},n}^\dagger \; t_{n,m}(\mathbf{i}-\mathbf{j}) c_{\mathbf{j},m}$$

$$\hat{K}_\alpha = \sum_{\mathbf{i},\mathbf{j},m.n} (\mathbf{j}-\mathbf{i})_\alpha^2 \, c_{\mathbf{i},n}^\dagger \; t_{n,m}(\mathbf{i}-\mathbf{j}) c_{\mathbf{j},m}$$

We are interested in the real part of the conductivity tensor.

$$\mathrm{Re}\,\sigma_{\alpha,\beta}(\omega) = -\frac{4\pi^2 c^2}{\Phi_0^2} \pi\delta(\omega) \overbrace{\frac{1}{N} \left\{ \left\langle K_\alpha \right\rangle_0 \delta_{\alpha,\beta} + \mathrm{Re}\,\Lambda_{\alpha\beta}(\omega = 0) \right\}}^{D_{\alpha\beta}} + \frac{4\pi^2 c^2}{\Phi_0^2} \frac{1}{N} \frac{\mathrm{Im}\,\Lambda_{\alpha\beta}(\omega = 0)}{\omega}$$

D_{xx} is the Drude weight.

Explicit calculation for an n-band non-interacting system. (FL Fixpoint)

$$\hat{H}_0 = \sum_{\mathbf{i},\mathbf{j},m.n} c^\dagger_{\mathbf{i},n}\; t_{n,m}(\mathbf{i}-\mathbf{j}) c_{\mathbf{j},m} = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} H(\mathbf{k}) c_{\mathbf{k}}, \qquad H(\mathbf{k}) = \sum_{\mathbf{r}} t(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\hat{\mathbf{J}}_P = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \left[\frac{\partial}{\partial \mathbf{k}} H(\mathbf{k}) \right] c_{\mathbf{k}} \equiv \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}}\; \mathbf{J}(\mathbf{k})\; c_{\mathbf{k}}$$

$$\hat{K}_\alpha = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \left[-\frac{\partial^2}{\partial k_\alpha^2} H(\mathbf{k}) \right] c_{\mathbf{k}} \equiv \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} K_\alpha(\mathbf{k}) c_{\mathbf{k}}$$

$H(\mathbf{k})$, $J_\alpha(\mathbf{k})$, $K_\alpha(\mathbf{k})$ are matrices of dimension # bands X # bands

With: $G_{n,m}(\mathbf{k}) = \left\langle c^\dagger_{\mathbf{k},m} c_{\mathbf{k},n} \right\rangle$

$$\Lambda_{\alpha\beta}(\omega) = i\int_0^\infty dt e^{i(\omega+i\eta)t} \sum_{\mathbf{k}} Tr\left[G(\mathbf{k}) \left[J_\alpha(\mathbf{k}),\; e^{-itH(\mathbf{k})} J_\beta(\mathbf{k}) e^{itH(\mathbf{k})} \right]\right]$$

Consequence: For a single band model, the above quantity vanishes. → $D_{xx} \propto \left\langle K_x \right\rangle \propto \frac{n}{m^*}$

Diagonalizing $H(\mathbf{k})$

$$U^\dagger(\mathbf{k})H(\mathbf{k})U(\mathbf{k}) \;=\; \mathrm{diag}(\,E_1(\mathbf{k}),\cdots,E_{\#bands}(\mathbf{k})\,)$$

$$H_{r,s}(\mathbf{k}) = \sum_n E_n(\mathbf{k})\,\underbrace{U_{r,n}(\mathbf{k})U^\dagger_{ns}(\mathbf{k})}_{=\left(P_n(\mathbf{k})\right)_{rs}}$$

$$H(\mathbf{k}) = \sum_n E_n(\mathbf{k})P_n(\mathbf{k}), \qquad P_n(\mathbf{k})P_m(\mathbf{k}) \;=\; P_n(\mathbf{k})\delta_{n,m}$$

$$e^{-iH(\mathbf{k})t} = \sum_n e^{-iE_n(\mathbf{k})t}P_n(\mathbf{k})$$

$$G(\mathbf{k}) = \sum_n f(E_n(\mathbf{k}))P_n(\mathbf{k}),$$

$$f(E_n(\mathbf{k})) = \frac{1}{e^{\beta(E_n(\mathbf{k})-\mu)}+1}$$

$$\sigma_{\alpha,\beta}(\omega) = \frac{4\pi^2c^2}{\Phi_0^2}\frac{1}{i(\omega+i\eta)}\frac{1}{N}\left[\sum_{n,\mathbf{k}} f\left(E_n(\mathbf{k})\right)\mathrm{Tr}\left[K_\alpha(\mathbf{k})P_n(\mathbf{k})\right]\delta_{\alpha,\beta} + \Lambda_{\alpha\beta}(\omega)\right]$$

$$\Lambda_{\alpha\beta}(\omega) = -\sum_{n,m,\mathbf{k}} \frac{f\left(E_m(\mathbf{k})\right)-f\left(E_n(\mathbf{k})\right)}{\omega+i\eta+E_m(\mathbf{k})-E_n(\mathbf{k})}\;\mathrm{Tr}\left[J_\alpha(\mathbf{k})P_n(\mathbf{k})J_\beta(\mathbf{k})P_n(\mathbf{k})\right]$$

$$\mathbf{J}(\mathbf{k}) = \frac{\partial}{\partial\mathbf{k}}H(\mathbf{k})$$

$$K_\alpha(\mathbf{k}) = -\frac{\partial^2}{\partial k_\alpha^2}H(\mathbf{k})$$

Two bands. Most general form for $H(\mathbf{k})$:

Source.

PHYSICAL REVIEW B **74**, 085308 (2006)

Topological quantization of the spin Hall effect in two-dimensional paramagnetic semiconductors

Xiao-Liang Qi,[1] Yong-Shi Wu,[2,1] and Shou-Cheng Zhang[3,1]

$$H(\mathbf{k}) = \varepsilon(\mathbf{k})\sigma_0 + V\,\mathbf{d}(\mathbf{k})\bullet\boldsymbol{\sigma}$$

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \boldsymbol{\sigma} = \left(\sigma^{(1)}, \sigma^{(2)}, \sigma^{(3)}\right) = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right)$$

$$H(\mathbf{k}) = E_+(\mathbf{k})P_+(\mathbf{k}) + E_-(\mathbf{k})P_-(\mathbf{k})$$

$$E_\pm(\mathbf{k}) = \varepsilon(k) \pm V\left|\mathbf{d}(\mathbf{k})\right|, \quad P_\pm(\mathbf{k}) = \frac{1}{2}\left(1 \pm \hat{\mathbf{d}}(\mathbf{k})\bullet\boldsymbol{\sigma}\right), \quad \hat{\mathbf{d}}(\mathbf{k}) = \mathbf{d}(\mathbf{k})/\left|\mathbf{d}(\mathbf{k})\right|$$

Calculation of $\sigma_{xx}(\omega)$ Drude weight.

$$D_{xx} = -\frac{1}{N}\sum_{n,\mathbf{k}} f\left(E_n(\mathbf{k})\right)\mathrm{Tr}\left[K_x(\mathbf{k})P_n(\mathbf{k})\right] + \frac{1}{N}\mathrm{Re}\,\Lambda_{xx}(\omega \to 0)$$

$$\frac{1}{N}\mathrm{Re}\,\Lambda_{xx}(\omega \to 0) = \frac{2}{N}\sum_{\mathbf{k}} \frac{f\left(E_-(\mathbf{k})\right) - f\left(E_+(\mathbf{k})\right)}{E_+(\mathbf{k}) - E_-(\mathbf{k})}\ \mathrm{Tr}\left[J_x(\mathbf{k})P_+(\mathbf{k})J_x(\mathbf{k})P_-(\mathbf{k})\right]$$

For the insulating case:

$$f\left(E_-(\mathbf{k})\right) = 1, \quad f\left(E_+(\mathbf{k})\right) = 0, \quad E_+(\mathbf{k}) - E_-(\mathbf{k}) = 2V\,|\,\mathbf{d}(\mathbf{k})\,| > 0 \quad \forall \;\; \mathbf{k}$$

and $$\lim_{N\to\infty} \frac{1}{N} \operatorname{Re}\Lambda_{xx}(\omega \to 0) = -\lim_{N\to\infty} \frac{1}{N} \sum_{\mathbf{k}} Tr\left[K_x(\mathbf{k}) P_-\right]$$

Hence the Drude weight vanishes and:

$$\operatorname{Re}\,\sigma_{x,x}(\omega) = \frac{4\pi^2 c^2}{\Phi_0^2} \frac{1}{N} \frac{\operatorname{Im}\Lambda_{xx}(\omega)}{\omega} = \frac{4\pi^2 c^2}{\Phi_0^2} \frac{\pi}{\omega} \underbrace{\frac{1}{N} \sum_{\mathbf{k}} \delta(\omega - 2V\,|\,\mathbf{d}(\mathbf{k})\,|) \;\; Tr\left(J_x P_- J_x P_+\right)}$$

Interband transitions.

→ $\operatorname{Re}\,\sigma_{x,x}(\omega)$ vanishes for frequencies below the charge gap $\min_k 2V|d(k)|$

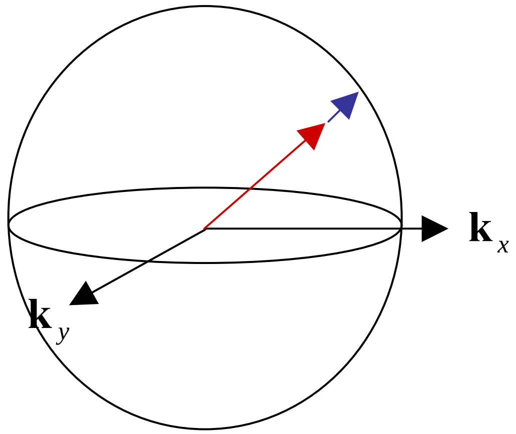
For the insulator:

$$\lim_{\omega\to 0}\sigma_{xy}(\omega) = -\frac{4\pi^2 c^2}{\Phi_0^2}\frac{1}{8\pi^2}\int_{BZ} d\mathbf{k}\,\underbrace{\left(\; f(E_-(\mathbf{k})) - f(E_+(\mathbf{k}))\;\right)}_{=1 \text{ for insulator}}\hat{\mathbf{d}}(\mathbf{k})\bullet\left(\left(\frac{\partial}{\partial k_x}\hat{\mathbf{d}}(\mathbf{k})\right)\times\left(\frac{\partial}{\partial k_y}\hat{\mathbf{d}}(\mathbf{k})\right)\right)$$

$$\int_{BZ} d\mathbf{k}\;\hat{\mathbf{d}}(\mathbf{k})\bullet\left(\left(\frac{\partial}{\partial k_x}\hat{\mathbf{d}}(\mathbf{k})\right)\times\left(\frac{\partial}{\partial k_y}\hat{\mathbf{d}}(\mathbf{k})\right)\right) = \pm 4\pi n$$

Where n is the number of times $\hat{\mathbf{d}}(\mathbf{k})$ winds around the unit sphere: topological quantity.

Examples a) n = 1 $\hat{\mathbf{d}}(\mathbf{k})$: one to one mapping between BZ and Unit sphere.



$$dk_x dk_y\left(\frac{\partial}{\partial k_x}\hat{\mathbf{d}}(\mathbf{k})\right)\times\left(\frac{\partial}{\partial k_y}\hat{\mathbf{d}}(\mathbf{k})\right) = d\mathbf{\Omega}$$

Oriented surface element of the sphere.

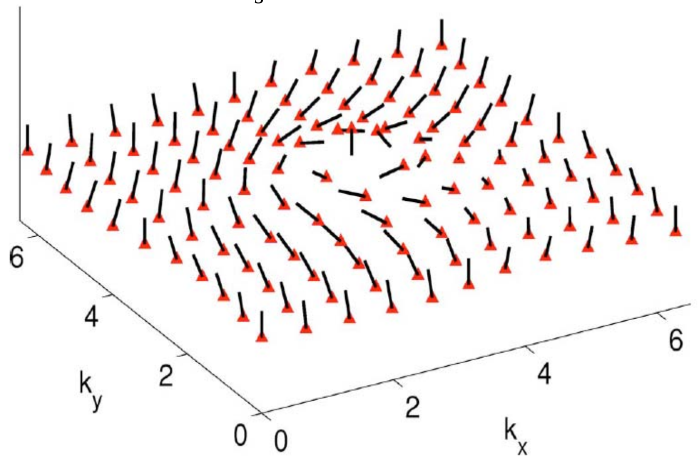
→Assume that orientation remains constant (outwards/inwards) for all BZ points.

$$\int_{BZ} d\mathbf{k}\;\hat{\mathbf{d}}(\mathbf{k})\bullet\left(\left(\frac{\partial}{\partial k_x}\hat{\mathbf{d}}(\mathbf{k})\right)\times\left(\frac{\partial}{\partial k_y}\hat{\mathbf{d}}(\mathbf{k})\right)\right) = \int_{S^2}\hat{\mathbf{d}}\bullet d\mathbf{\Omega} = \pm 4\pi$$

$$\varepsilon(\mathbf{k}) = -2t(\cos(\mathbf{k}_x) + \cos(\mathbf{k}_y))$$

$$\mathbf{d}(\mathbf{k}) = (\sin(\mathbf{k}_y), -\sin(\mathbf{k}_x), c(2-\cos(\mathbf{k}_x)-\cos(\mathbf{k}_y)-e_s))$$

$e_s = 3.7$



b) $e_s < 0$ $\hat{\mathbf{d}}(\mathbf{k})$ covers a surface on the northern hemisphere. Since $\hat{\mathbf{d}}(\mathbf{k}+\mathbf{b}_i) = \hat{\mathbf{d}}(\mathbf{k})$ this surface is covered twice. Once with orientation $\hat{\mathbf{d}}\bullet d\mathbf{\Omega} > 0$ and once with orientation $\hat{\mathbf{d}}\bullet d\mathbf{\Omega} < 0$

The integral hence vanishes.

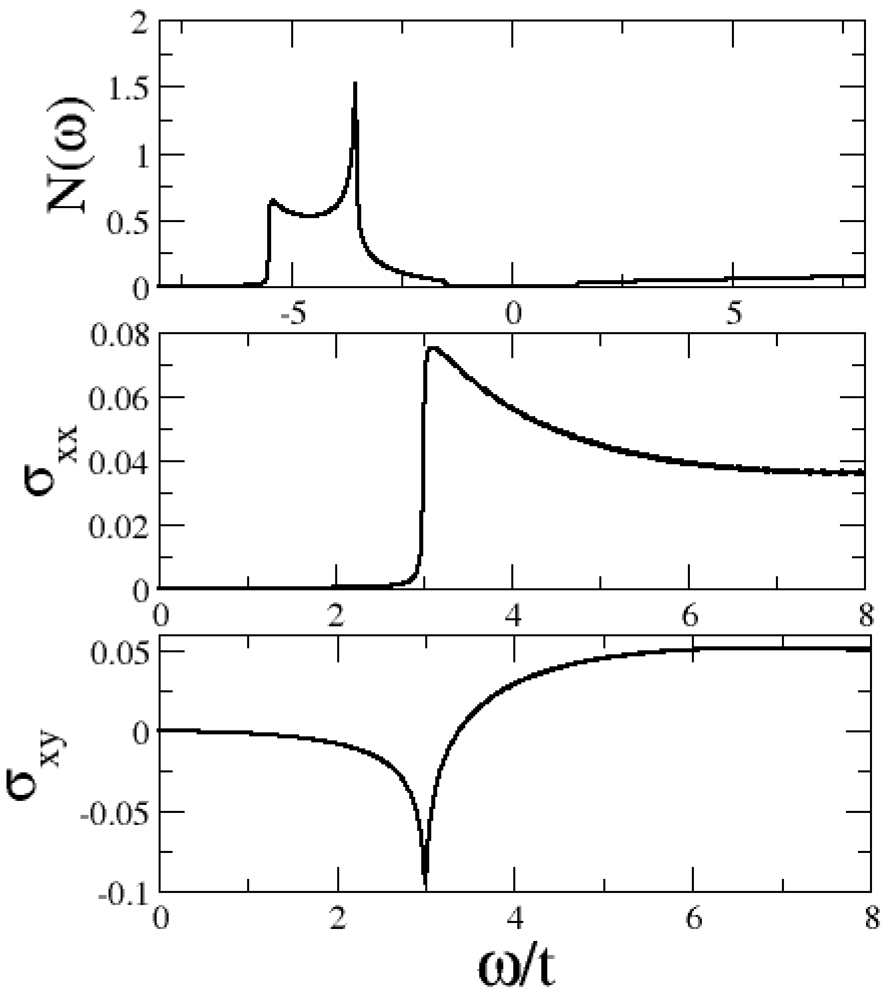
<u>In general.</u>

$$\lim_{\omega\to 0} \sigma_{x,y}(\omega) = \frac{4\pi^2 c^2}{\Phi_0^2} \begin{cases} 0 & e_s > 4,\ e_s < 0 \\ \dfrac{1}{2\pi} & 2 > e_s > 0 \\ -\dfrac{1}{2\pi} & 4 > e_s > 2 \end{cases}$$

Optical conductivity as a function of e_s.

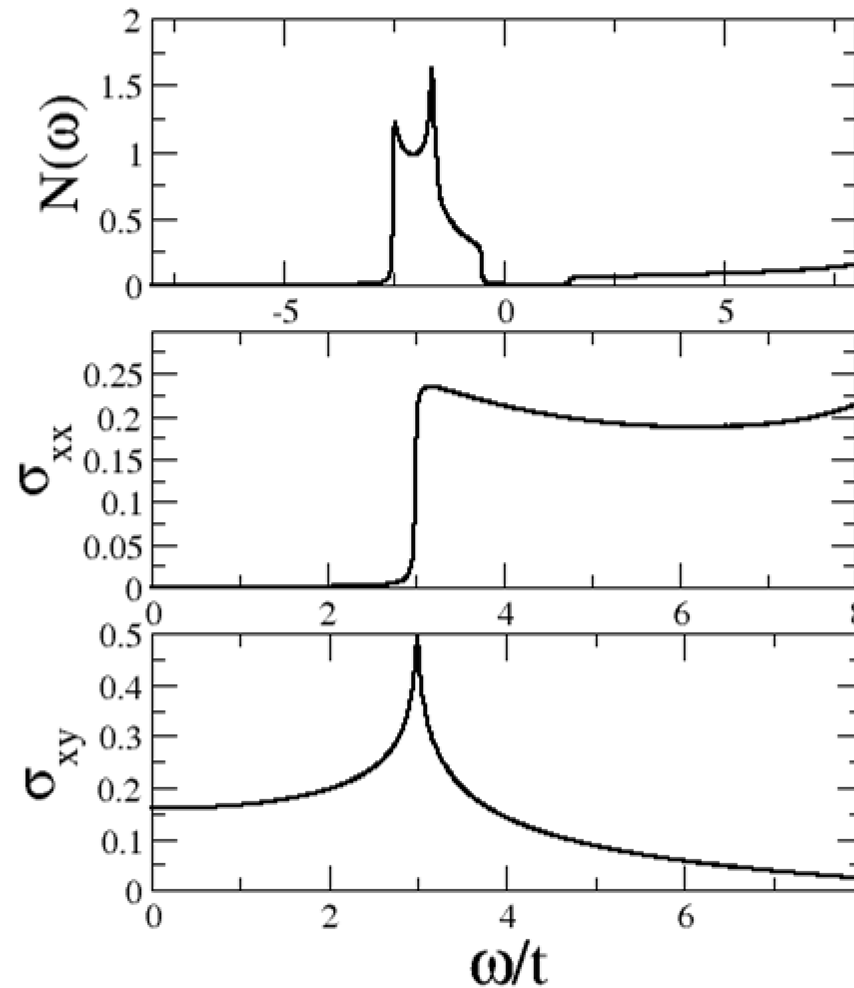
Insulator.

es=-0.5



Topological Insulator.

es=0.5



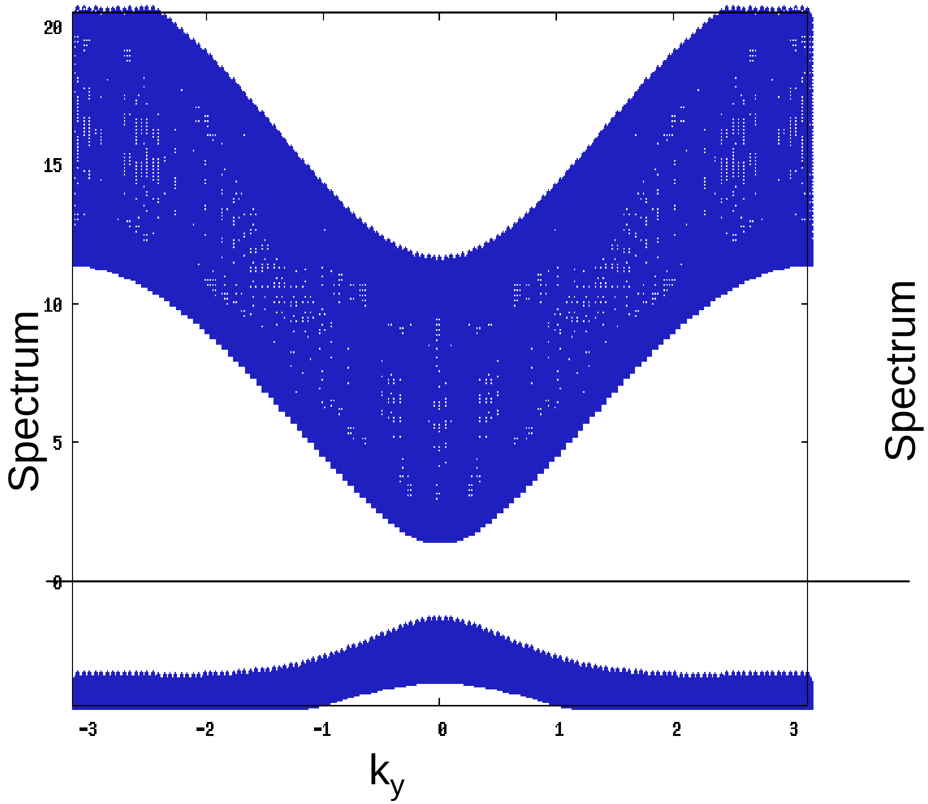
$$\frac{4\pi^2 c^2}{\Phi_0^2} = 1$$

Edge states. Insert infinite potential on sites $(i_x, i_y = L)$ → wave function vanishes on the edge.

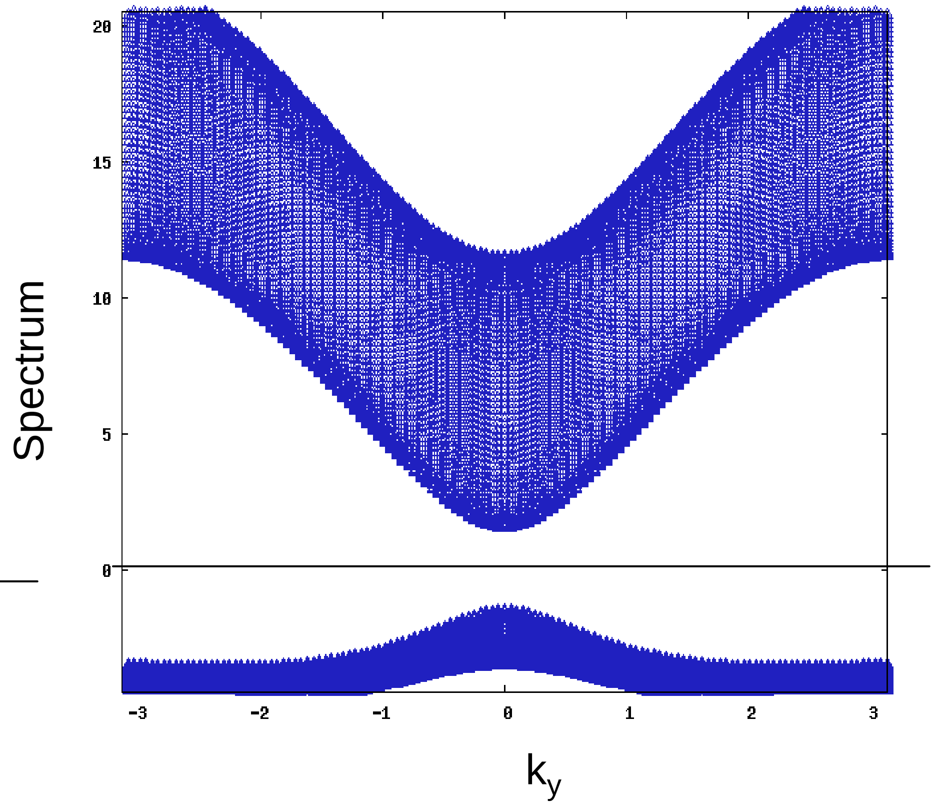
→ k_y is still a good quantum number. Hamiltonian is block diagonal and k_y labels the blocs. For each k_y we have 2L energy states.

Insulator: e_s=-0.5

Open in x

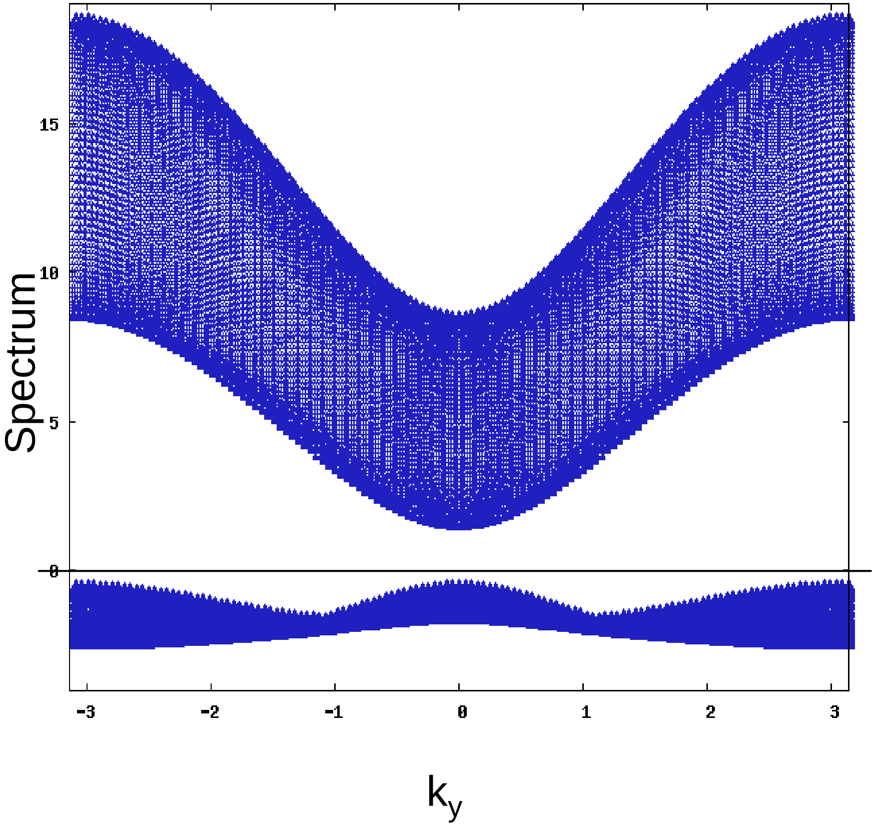


Periodic in x

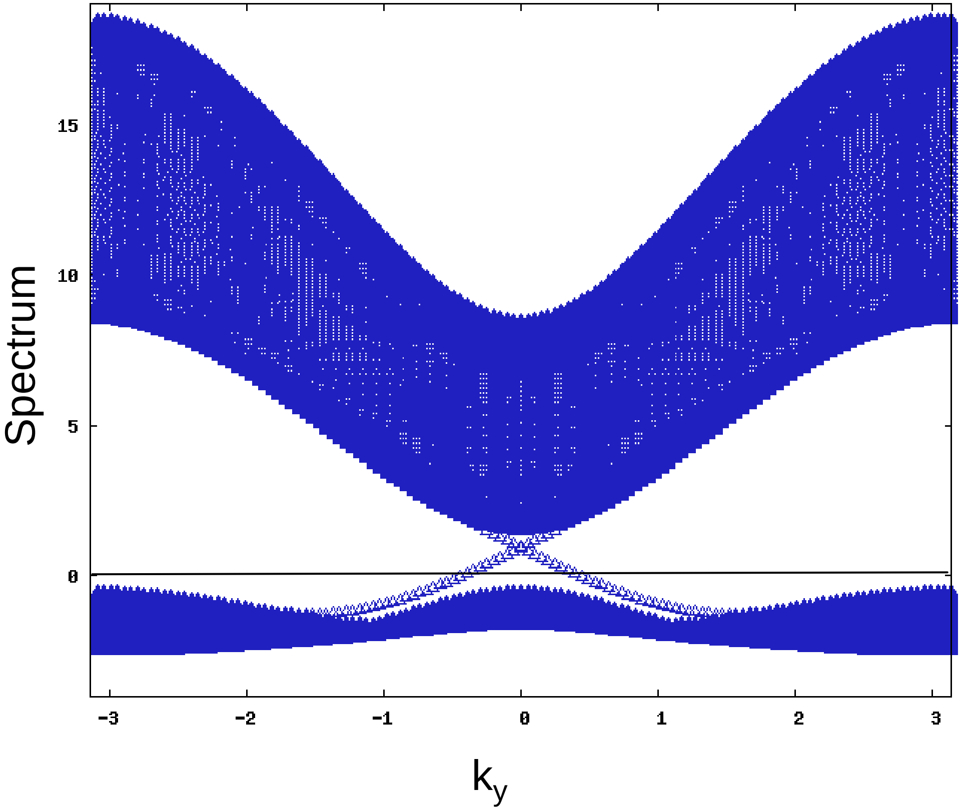


Topological Insulator: e_s=0.5

Periodic

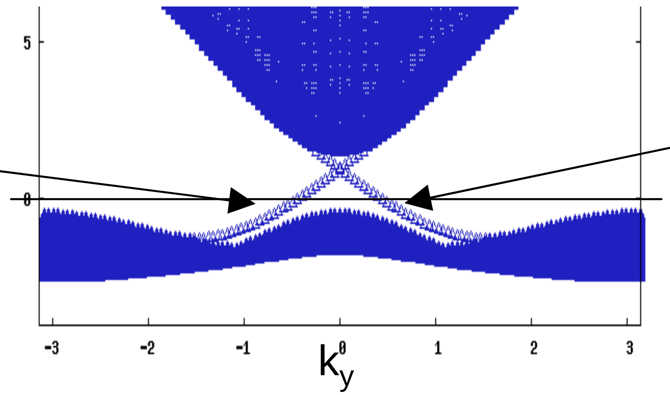


Open.



Gapless excitations are located on the edge of the sample. To see this consider the i_x and k_y resolved density of states:

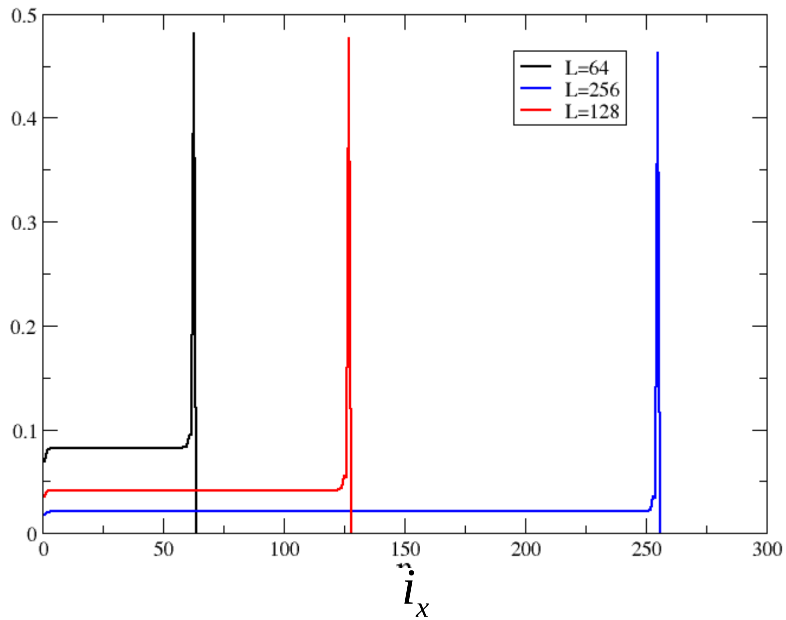
Flows on the rhs of sample.
$v_y > 0$

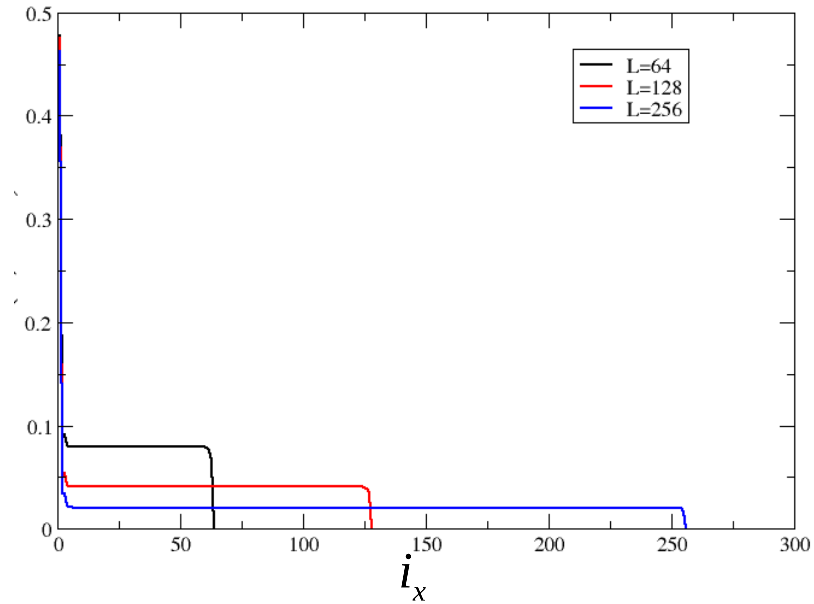


Flows on the lhs of sample.
$v_y < 0$

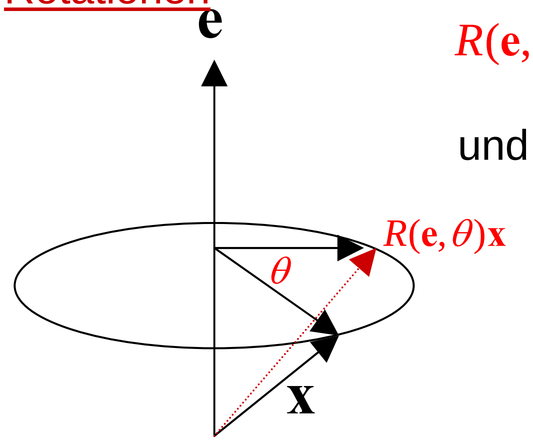
$$N^{-}(i_x, \omega = 0) = \frac{2}{L} \sum_{k_y < 0} N_{i_x, k_y}(\omega = 0)$$

$$N^{+}(i_x, \omega = 0) = \frac{2}{L} \sum_{k_y > 0} N_{i_x, k_y}(\omega = 0)$$





Rotationen



$$R(\mathbf{e},\theta)\mathbf{x} = \mathbf{e}\bullet(\mathbf{e}\times\mathbf{x}) - \cos(\theta)\,\mathbf{e}\times(\mathbf{e}\times\mathbf{x}) + \sin(\theta)\,\mathbf{e}\times\mathbf{x}$$

und es gilt: $$\frac{d}{d\theta}R(\mathbf{e},\theta)\mathbf{x} = \mathbf{e}\times R(\mathbf{e},\theta)\,\mathbf{x}$$

Damit ist: $R(\mathbf{e},\theta)\mathbf{x}$ ist Lösung der Dgl.

$$\frac{d}{d\theta}\mathbf{x}(\theta) = \mathbf{e}\times\mathbf{x}(\theta) \quad \text{mit} \quad \mathbf{x}(\theta=0) = \mathbf{x}$$

Spin Operator, $\hat{\mathbf{S}}$, ist Erzeuger von Rotationen.

Sei $$\hat{T}(\mathbf{e},\Theta) = e^{-i\mathbf{e}\bullet\hat{\mathbf{S}}\Theta/\hbar}$$

Dann gilt: $$\hat{\mathbf{S}}(\Theta) \equiv \hat{T}^{\dagger}(\mathbf{e},\Theta)\,\hat{\mathbf{S}}\,\hat{T}(\mathbf{e},\Theta) = R(\mathbf{e},\Theta)\,\hat{\mathbf{S}}$$

da aus: $$\left[\hat{\mathbf{S}}_{\alpha},\hat{\mathbf{S}}_{\beta}\right] = i\varepsilon^{\alpha\beta\gamma}\hat{\mathbf{S}}_{\gamma}$$

folgt: $$\frac{d}{d\theta}\hat{\mathbf{S}}(\theta) = \mathbf{e}\times\hat{\mathbf{S}}(\theta)$$

Further reading.

On the relation between the topological bulk number and edge states !

A General Theorem Relating the Bulk Topological Number to Edge States in Two-dimensional Insulators

Xiao-Liang Qi[1,2], Yong-Shi Wu[3] and Shou-Cheng Zhang[2,1]

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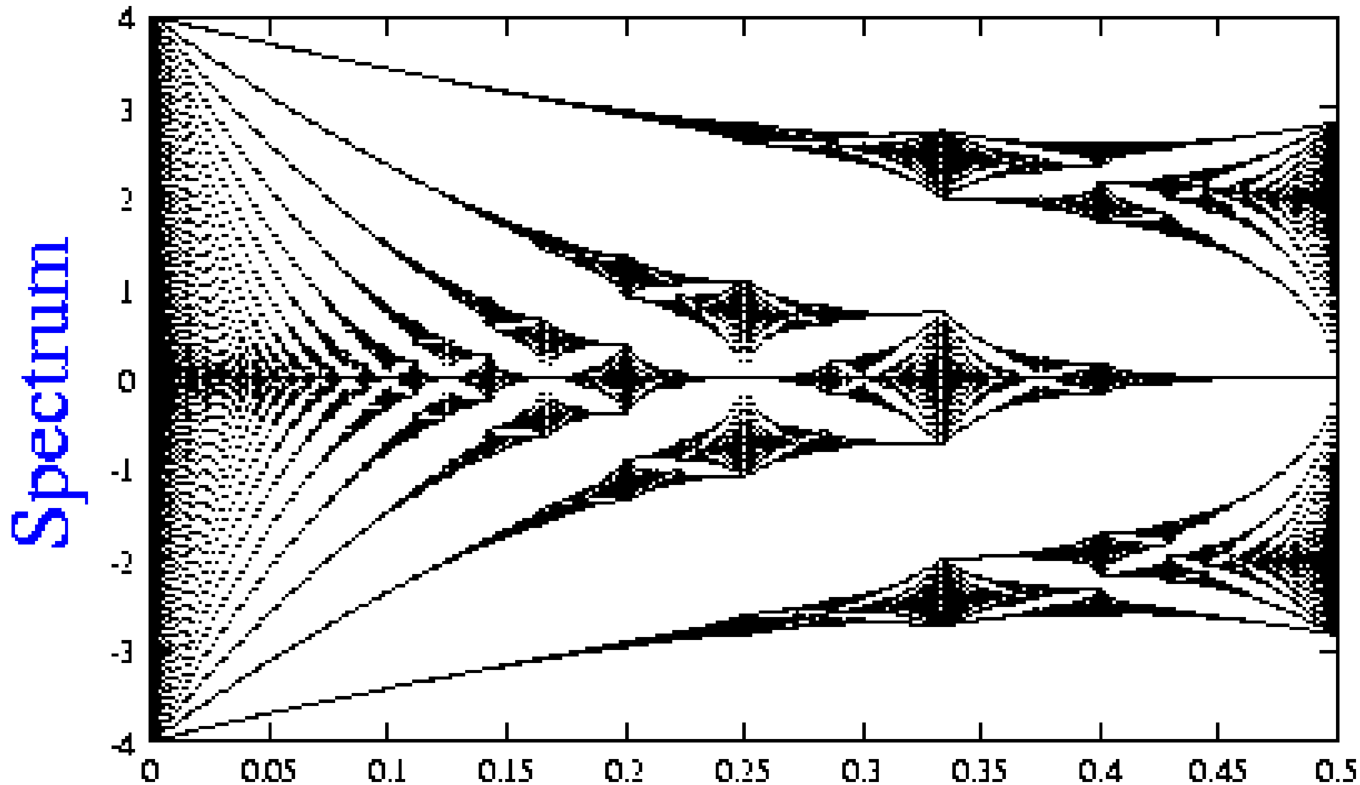
(Dated: October 28, 2009)

arXiv:cond-mat/0604071v2 [cond-mat.mes-hall] 5 Apr 2006

L = 24


Spectrum
4
3
2
1
0
-1
-2
-3
-4
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Magnetic flux / plaquette