Neutrino Oscillations and Mass.

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- Introduction.

  The building blocks of Matter

- The Experiment.

  Oscillations of Atmospheric Neutrinos

- The Weinberg-Salam Model.

  Can it incorporate neutrino mass?

$\boxed{\text{Building blocks of Matter: } (S = 1/2)\text{Fermions:}}$

$$\text{QUARKS} \quad \begin{pmatrix} u & u & u \\ d & d & d \end{pmatrix} \begin{pmatrix} s & s & s \\ c & c & c \end{pmatrix} \begin{pmatrix} t & t & t \\ b & b & b \end{pmatrix}$$

$$\text{LEPTONS} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

$\boxed{\text{They interact via Gauge Bosons:}}$

- The Photon.

  couples to electric charge. U(1) gauge symmetry

- The W^+, W^-, Z_0.

  Mediate the weak interactions. SU(2) gauge invaraince

- 8 Gluons

  Couple to the color degrees of freedom of the quarks. SU(3) gauge invaraince

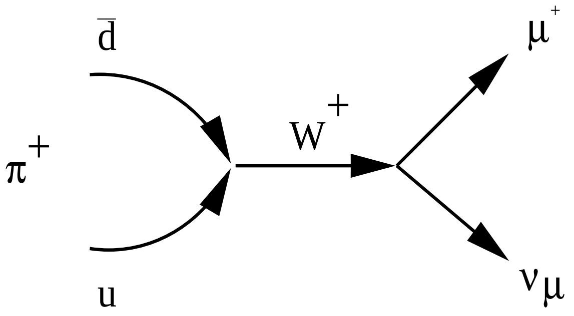
$\boxed{\text{Mass of Gauge Bosons and Fermions:}}$

- Higgs Field: h(x)

$\boxed{\text{The experiment.}}$

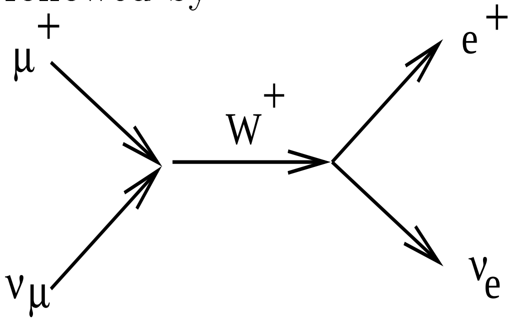
- <u>Neutrino Source. Atmospheric Neutrinos</u>

Interaction of cosmic rays with nuclei in upper atmosphere produce hadron showers. Production of atmospheric neutrino dominated by:



$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

followed by



$$\mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

In the range $0.1 GeV$ to $10 GeV$:

$$\boxed{\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} \sim 2 \pm 5\%}$$

M. Honda et al. Phys. Lett. **B 248**, 193 (1990)

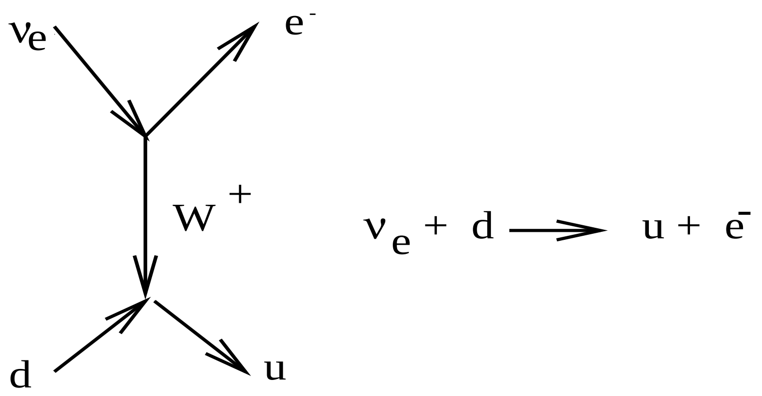
M. Honda et al. Phys. Lett. **D52**, 4985 (1995).

• The Super-Kamiokande Detector

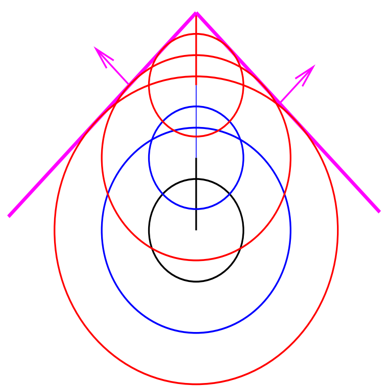
⋆ 50'000 tons H_2O underground detector.
⋆ 13'000 photomultiplier on side, top and bottom facing the inner volume.
⋆ Neutrinos are detected with:

$$\boxed{\nu + N \to K + l}$$

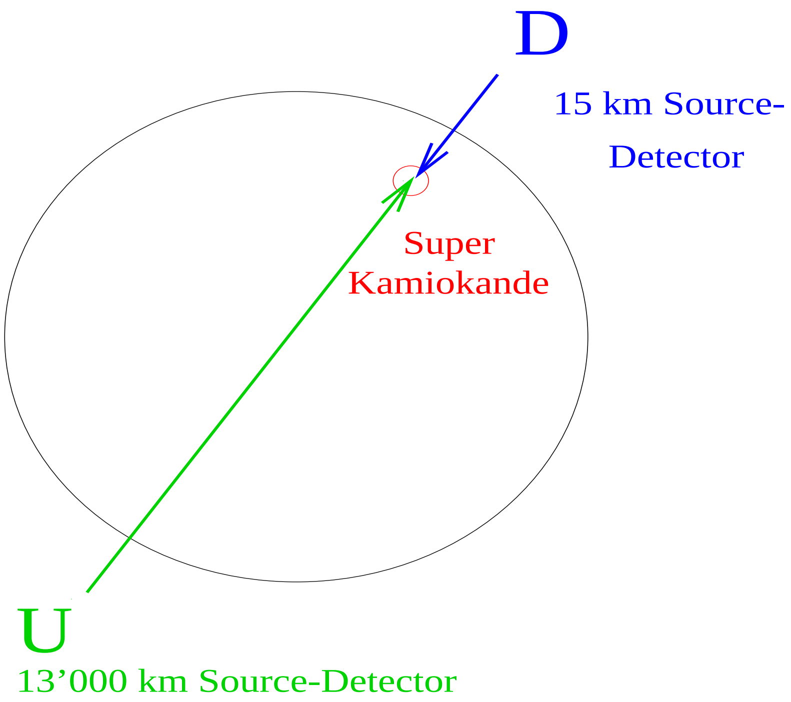
N, K nuclei. l: lepton, ν is neutrino



Detect lepton through it's Cherenkov radiation:
Charged particle moving in a medium emits light by exciting surrounding atoms. If the speed of the charged particle, βc, is greater than the speed of light in the medium, c/n, the light emitted by excited atoms appears in the form of a coherent wavefront.

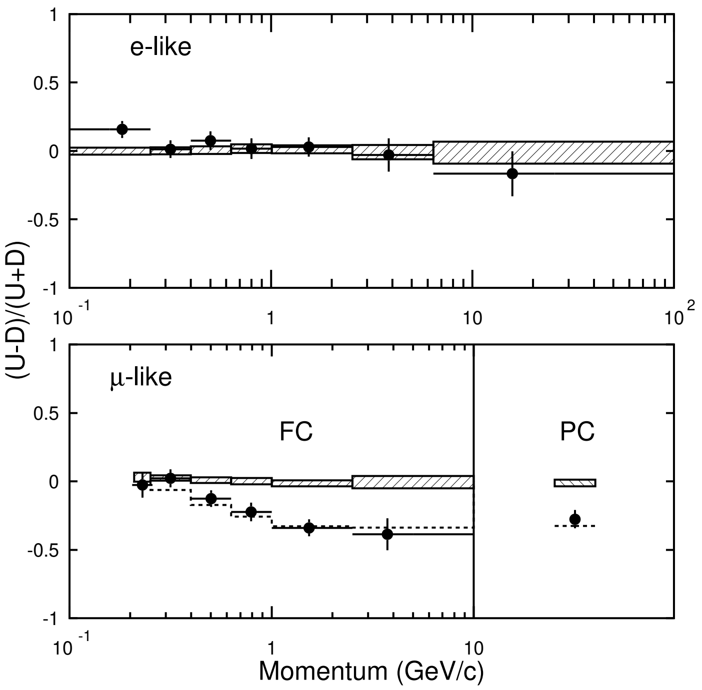


- <u>The Measured quantity</u>



Measure

$$\frac{U - D}{U + D}$$



The Super-Kamikande collaboration Phys.Rev.Lett. 81 (1998) 1562-1567

- Interpretation

The μ-neutrinos are mixtures of states with different masses.

Two Neutrino Model.

$$\begin{pmatrix} |\nu_\mu\rangle \\ |N\rangle \end{pmatrix} = \begin{pmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}, \quad H|\nu_i\rangle = E_i|\nu_i\rangle$$

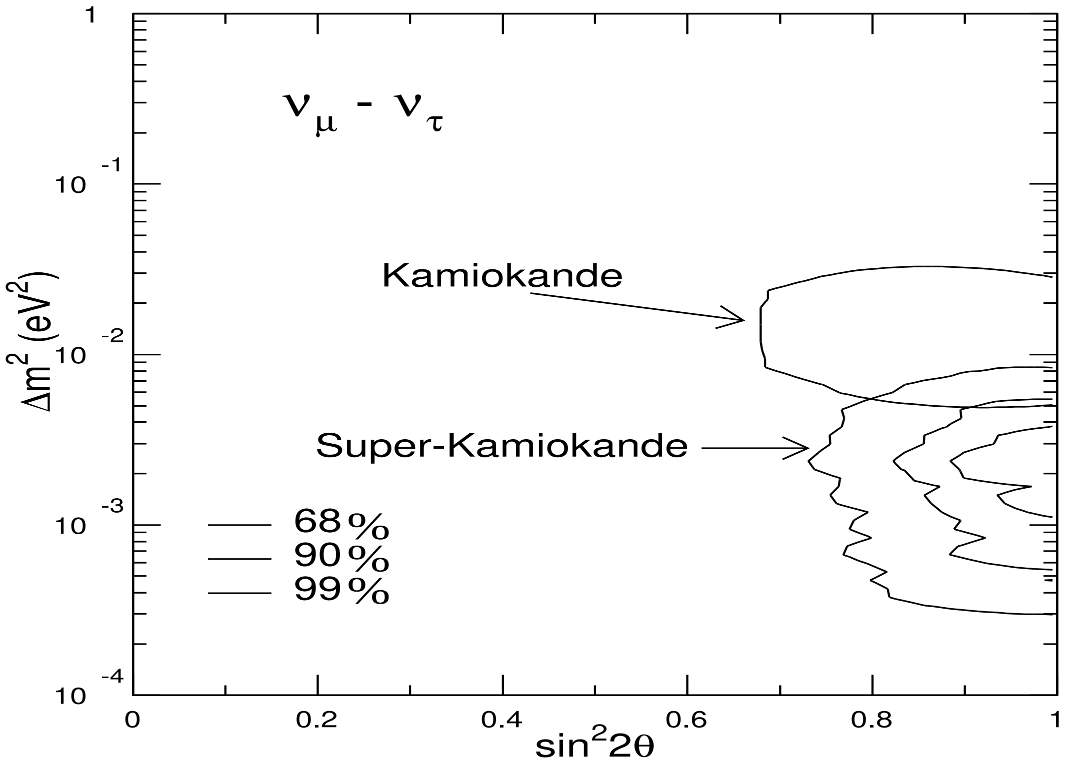
$$E_i = \sqrt{|\vec{p}_i|^2 + m_i^2} \to |\vec{p}| + m_i^2/2|\vec{p}|$$

$\vec{p}_i = \vec{p}$ and $E_i \gg m_i$

$$\begin{pmatrix} |\nu_\mu(t)\rangle \\ |N(t)\rangle \end{pmatrix} = \begin{pmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{pmatrix} \begin{pmatrix} e^{iE_1 t}|\nu_1\rangle \\ e^{iE_2 t}|\nu_2\rangle \end{pmatrix}$$

$$\boxed{|\langle\nu_\mu(t)|\nu_\mu(0)\rangle|^2 = 1 - \sin^2(2\Theta)\sin^2\left(\frac{\Delta m^2 L}{p}\right)}$$

$$\boxed{\Delta m^2 = m_1^2 - m_2^2}$$



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At 90% confidence level,

$$\boxed{\begin{array}{c} \sin^2(2\Theta) > 0.82 \\ 5 \times 10^{-4} < \Delta m^2 < 6 \times 10^{-3} eV^2 \end{array}}$$

$\boxed{\nu_\mu \text{ and } \nu_\tau \text{ have different masses.}}$

The Standard Model

Ingredients of the Standard Model.

Weinberg-Salam model (1967)

- Description of Spin 1/2 particles.

  Dirac Equation.

- Specific multiplets and charges.

- Gauge principles.

  $SU(2) \otimes U(1) \otimes SU(3)$ local gauge symmetry.

- Higgs mechanism.

  Spontaneous symmetry breaking $\rightarrow$ mass for vector bosons and fermions.

Spontaneously broken non-abelian gauge theories are renormalizable (t'Hooft 1971).

By cleverly redefining fields, masses and couplings one can absorb all divergences.

The Dirac Equation

$$\mathcal{L} = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - m\right)\psi(x)$$

$$x^\mu = (t, \vec{x}) \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}$$

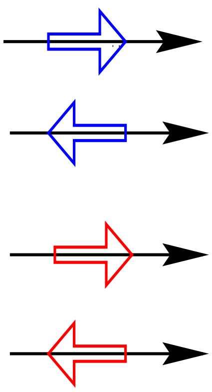
$$\gamma^\mu = \left[\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}\right] \quad \bar{\Psi} = \Psi^\dagger \gamma^0$$

Euler-Lagrange equations yield the Dirac Equation:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} - \frac{\partial \mathcal{L}}{\partial \bar{\psi}} = 0$$

$$\boxed{\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0}$$

The equation has four solutions with $E = \pm\sqrt{\vec{p}^2 + m^2}$.



Particles: $E > 0$

Ψ_1 Positive Helicity

Ψ_2 Negative Helicity

Antiparticles: $E < 0$

Ψ_3 Positive Helicity

Ψ_4 Negative Helicity

$$\boxed{\begin{array}{c} E = p^0 = \sqrt{\vec{p}^2 + m^2} \\[2em] \psi_1 = e^{-ipx}\left(\begin{array}{c} \chi^+(\vec{p}) \\ \frac{|\vec{p}|}{E+m}\chi^+(\vec{p}) \end{array}\right) \;\; \psi_2 = e^{-ipx}\left(\begin{array}{c} \chi^-(\vec{p}) \\ \frac{-|\vec{p}|}{E+m}\chi^-(\vec{p}) \end{array}\right) \\[2em] E = p^0 = -\sqrt{\vec{p}^2 + m^2} \\[2em] \psi_3 = e^{-ipx}\left(\begin{array}{c} \frac{-|\vec{p}|}{E+m}\chi^+(\vec{p}) \\ \chi^+(\vec{p}) \end{array}\right) \;\; \psi_4 = e^{-ipx}\left(\begin{array}{c} \frac{|\vec{p}|}{E+m}\chi^-(\vec{p}) \\ \chi^-(\vec{p}) \end{array}\right) \end{array}}$$

with

$$\vec{\sigma} \cdot \vec{p}\chi^{\pm}(\vec{p}) = \pm\chi^{\pm}(\vec{p}).$$

Helicity operator:

$$\Sigma = \frac{1}{2}\left(\begin{array}{cc} \vec{\sigma} \cdot \hat{\vec{p}} & 0 \\ 0 & \vec{\sigma} \cdot \hat{\vec{p}} \end{array}\right)$$

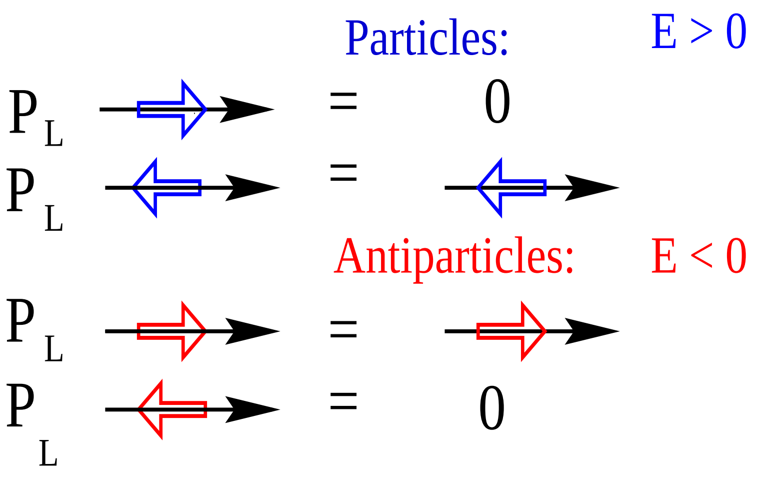
Left-Handed, Right-Handed.

$$\boxed{\begin{aligned} \psi_L = P_L\psi &\equiv \tfrac{1}{2}(1-\gamma^5)\psi \\ \\ \psi_R = P_R\psi &\equiv \tfrac{1}{2}(1+\gamma^5)\psi \end{aligned}}$$

$$\boxed{\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \equiv i\gamma^0\gamma^1\gamma^2\gamma^3}$$

$P_R + P_L = 1$, $P_R P_L = 0$, $(P_L)^2 = P_L$ and $(P_R)^2 = P_R$

In relativistic limit $|E| >> m$ or $m = 0$



Thus, a massless particle satisfies

$$\boxed{[H_D, P_R] = [H_D, P_L] = 0}$$

handedness is a conserved quantity.

The multiplet structure of the Standard Model.

- Start with the massless left-handed and right-handed spinors. i.e. e_R is a massless right-handed electron.

  Mass will be generated by interaction with the Higgs Field

- Weak interactions break parity and couple only to left-handed particles. Under parity: $P_R \rightarrow P_L$

- Organize the particles of a given family into $SU(2)$ weak isospin and $U(1)$ weak hypercharge.

Weak isospin doublet and weak hyperchage.

Lepton	T	T_3	Y	$Q = T_3 + \frac{Y}{2}$
$\nu_{e,L}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	0
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e_R	0	0	-2	-1

Quark	T	T_3	Y	$Q = T_3 + \frac{Y}{2}$
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$
u_R	0	0	$\frac{4}{3}$	$\frac{2}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$

- Weak interactions couple to SU(2) Weak Isospin T
- Electromagnetic interactions couple to $U(1)$ Electric charge Q

- There is a total of 15 particles. (Including the color degrees of freedom of quarks.)

How do the particles interact?

• $SU(2) \otimes U(1)$ local gauge invariance. Leads to conservation of weak isospin and weak hypercharge (Noether.)

Let.

$$\chi_L = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \text{ or } \begin{pmatrix} u_L \\ d_L \end{pmatrix} \cdots$$

and

$$\Psi_R = e_R \text{ or } u_R \cdots$$

Those particle are described by the Dirac equation:

$$\mathcal{L} = \bar{\chi}_L(x) \left(i\gamma^\mu \partial_\mu \right) \chi_L(x) + \bar{\Psi}_R \left(i\gamma^\mu \partial_\mu \right) \Psi_R$$

Require that above equation is invariant under

$$\boxed{\begin{array}{c} \chi_L(x) \to e^{iY\beta(x)} e^{i\vec{T}\vec{\alpha}(x)} \chi_L(x) \\ \\ \Psi_R(x) \to e^{iY\beta(x)} \Psi_R(x) \end{array}}$$

Possible by introducing gauge bosons which transform as:

$$\boxed{\begin{array}{c} B_\mu(x) \to B_\mu(x) - \frac{1}{g}\partial_\mu \beta(x) \\ \\ \vec{W}_\mu(x) \to \vec{W}_\mu(x) - \frac{1}{g'}\partial_\mu \vec{\alpha}(x) - \vec{\alpha}(x) \times \vec{W}_\mu(x) \end{array}}$$

and

$$\boxed{\begin{array}{c} \mathcal{L} = \bar{\chi}_L(x) \gamma^\mu \left[i\partial_\mu - g'\vec{T}\vec{W}_\mu(x) - gY B_\mu(x) \right] \chi_L(x) + \\ \\ \bar{\Psi}_R \gamma^\mu \left[i\partial_\mu - gY B_\mu(x) \right] \Psi_R \end{array}}$$

• How do the particles acquire mass?

Higgs mechanism.

Problem:

$$\mathcal{L}_e = -m_e \bar{e} e \equiv -m_e \left(\bar{e}_R e_L + \bar{e_L} e_R \right)$$

is not gauge invariant.

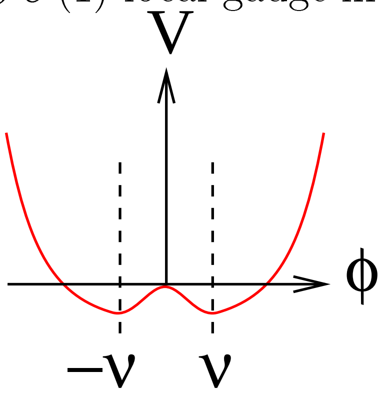
Solution: Introduce Higgs field

$$\boxed{\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}}$$

Φ: isospin doublet and hypercharge $Y = 1$.

$$\boxed{\mathcal{L}_e = -G_e \left[\bar{\chi}_L \Phi e_R + \bar{e}_R \bar{\Phi} \chi_L \right] + V(\Phi)}$$

Has $SU(2) \otimes U(1)$ local gauge invariance.



Spontaneous symmetry breaking:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\boxed{\mathcal{L}_e = -\frac{G_e v}{\sqrt{2}} \bar{e} e - G_e \sqrt{2} \bar{e} e h}$$

• Can the Neutrino acquire mass?

• There is no right-handed neutrino. Thus a Dirac mass term

$$-m_\nu \left(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R\right)$$

may not be generated with the Higgs mechanism.

• From the spinor of the left-handed neutrino ν_L, one can build a Majonara spinor ν_M. One may then include a Majonara mass term:

$$-m_\nu \left(\bar{\nu}_M \nu_M\right)$$

If inserted by hand, this term breaks gauge symmetry and the theory is not renormalizable.
Cannot be generated with a Higgs field with $Y = 1$.

The neutrino mass is beyond the framework of the standard model.

Just add the right handed neutrino?

Lepton	T	T_3	Y	$Q = T_3 + \frac{Y}{2}$
$\nu_{e,L}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	0
e_L	$\frac{1}{2}$	-$\frac{1}{2}$	-1	-1
$\nu_{e,R}$	0	0	0	0
e_R	0	0	-2	-1

Quark	T	T_3	Y	$Q = T_3 + \frac{Y}{2}$
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
d_L	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{2}{3}$	-$\frac{1}{3}$
u_R	0	0	$\frac{4}{3}$	$\frac{2}{3}$
d_R	0	0	-$\frac{1}{3}$	-$\frac{1}{3}$

Higgs mechanism: $\rightarrow$

$$-m_\nu \left(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R\right)$$

- But:

Now there is a total of 16 particles in a single family. Possible to classify them in a 16 dimensional spinor representation of $SO(10)$

F. Wilczek Nature **394**, 13, (1998).

Conclusions.

- Evidence for Neutrino Oscillations.
- Neutrinos have mass.
- The mass of the neutrino can not be taken into account in the framework of the standard model.