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Divergence of Wealth in a Segmented Society
Part I: 2-Segments Model
by
Dieter Roess

1.) Introduction

With his book *Capital in the Twenty-First Century*[1] Thomas Piketty has initiated a world wide dispute about the concentration of Capital within a small fraction of the population. He uses huge statistical evidence to demonstrate historic developments of wealth concentration. He describes basic mechanisms leading to it, and he presents proposals how to influence this process. As a *Fundamental Source of Divergence* he denominates[2] the relation $r > g$ (the profit of capital r being larger than the growth of income g).

As quite a number of parameters are interacting in the concentration of capital, which may change in time, his literal treatment is necessarily rather complicated and voluminous. For my better understanding I developed a mathematical analysis and numerical simulation *sine ira et studio* of the algorithms underlying the problem.

It is helpful in understanding the basic interrelations. The simulation delivers scenarios of the character *if A, B, C, and D are true, then*…Even if such a model does not claim to map reality in its complexity, it assists in quickly developing a judgement of what is important and what is not. Such a treatment extends the predicting power of Piketty´s fundamental relation to the determination of possible time functions of segregation and of critical states of its development.

In this paper an analysis is presented for the model of a society with 2 segments: *Have-Not* and *Have*.

2.) Statement of the Problem to be analyzed

Recently the time dependence of Income I and Capital C of a homogeneous society in dependence on constant growing and savings rate was derived and analyzed by the author[3] (a generalization of Piketty´s *Second Law*).

Now a society will be investigated that is segmented into groups with different initial capital, income, saving behaviour and opportunities to achieve a certain rate of return of their capital. This helps to clarify the role of Piketty´s *Fundamental Source of Divergence* and connects it with the time dependence of drifting apart in segmented societies.

In the present Part I just two segments are considered with unequal initial capital and income, but equal growth of income. Part II will treat a highly segmented society, under few restrictions.

All monetary values and all parameters stated are net, after tax, after cost, and deflated. It is assumed that all wealth in a society belongs to private persons; objects in the property of the state are balanced by debt of the state to private persons. Income of the state (taxes, etc) is immediately redistributed into private income.

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[1] Thomas Piketty *Capital in the Twenty-First Century* ,The Bellnap Press of Harvard University Press 2014, p.166
[2] P. 25 and p. 353 onward
[3] http://www.physik.uni-wuerzburg.de/~roess/Piketty.htm

For the following analysis capital is defined as wealth accumulated by inheritance, and by the saving of income that is not consumed but used for the generation of additional capital through profit.

Money saved is considered as retarded consumed income if it does not generate new capital and is accumulated for long lived consumable goods (e.g. car, self- used house, pensions, retirement income). In statistical data this is a part of *total capital*. In the present treatment this part of capital is to be deducted from the statistical value.

In the previous analysis of the *Second Law* the time change of Capital C versus Income I in a period of time $0 \le t \le T$ was defined as

$$C(t) = C(0) + \int_0^T \sigma(t) I(t) dt$$

with $C(0)$ the initial Capital and σ the savings rate of income I . For exponential growth of income with constant rate g and savings rate $\sigma(t) = s$

$$I(t) = I(0) e^{gt}$$

$$C(t) = C(0) + \frac{s}{g} I(0)(e^{gt} - 1)$$

$$\frac{C(t)}{I(t)} = \frac{C(0)}{I(0)} e^{-gt} + \frac{s}{g}(1 - e^{-gt})$$

This is the time dependent formulation of Piketty´s *Second Law of Capitalism.* The model treats the society as a homogeneous entity, with equal distribution of Capital and income. At realistically low growing rates ($g < 5\%$) and moderate savings rates ($s < 10\%$) changes in time are slow.

To understand the fast change in the distribution of income and wealth in modern capitalistic societies one must segregate the society into segments with different initial capital, income and saving behaviour. The deciding questions are:

Who gets the income from capital?
What do they do with it?

3.) Time development in a Society with 2 Segment: *Have* and *Have-Not*

For illustrating the principle we treat in this paper a highly simplified model of a society with just 2 segments.

- *Have (B)*
- *Have-not (A)*

Such a simple model has the charm that it can be treated analytically, deriving some useful formulas, while already visualizing basic trends of a more detailed numerical analysis. A beneficial feature of mathematical simulation is that its underlying assumptions are more explicit than in verbal argumentation.

Have (B):

- own all the interest bearing capital (which is a sizable part of total capital)
- Have a work income so high that it is plentiful for their desired living standard
- Therefore they can save and reinvest most or all of their capital income, and possibly also part of their work income

- They are a small percentage of the population

Have-Not (A):

- Own no capital in the sense of profit creating investment
- Have no capital income
- Spend their total income, currently or retarded.
- Are by far the greater parts of the population

As *capital income* we define profit on existing capital; it consists of dividends[4] and the value gain of investments[5] . The rest of total income is work income. It consists of wage, fee, reimbursement as manager, etc.

$$I(t) = I_C(t) + I_W(t)$$

With ρ the profit rate and σ the saving rate of B capital (both assumed constant in time)

$$I_C(t) = \rho C(t)$$

$$C(t) = C(0) + \sigma \int_0^T (I_W(t) + \rho C(t))dt \quad \text{differentiate}$$

$$\frac{dC(t)}{dt} = \sigma I_W(t) + \sigma\rho C(t)$$

The change of capital is due to the saving of B work and capital income.

Let us assume that work income is spent for living expenses; all savings are then stemming from capital income (saving of work income is considered in the simulation; it will not change the situation substantially). This allows deriving the time dependence of Capital as a simple formula by its differential equation, which turns out to be that of the exponential function

$$\rho C(B) \gg I_W(B) \rightarrow$$

$$\frac{dC(t)}{C(t)} = \rho\sigma dt \rightarrow$$

$$\ln C(t) = \rho\sigma t + C(0)$$

$$C(t) = C(0)e^{\rho\sigma t}$$

With constant savings and interest rate, capital increases exponentially in time, independent of the growing rate of total income *g*. Its growth parameter $\rho\sigma$ is the product of the profit rate of capital ρ and of the savings rate σ within the B segment. In time $1/\rho\sigma$ capital will increase by a factor e = 2.718[6]…

Im the total society the savings rate will be *s* << 100%; hence *rs* is very small. In the *B* segment *s* can be close to 1, at profit rate $\rho > r$ larger than for A. Increase of its capital can be correspondingly faster.

$$I_W(t) = I(t) - I_C(t) = I(t) - \rho C(0)e^{\rho\sigma t}$$

Only the residue of total income after deduction of capital income is left for work income. It will necessarily increase less than total income with $\rho C > 0$.

Now we assume an exponential growing model for total income[7] too:

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[4] In Germany dividends are taxed at the flat rate of 25% (*Abgeltungssteuer*)

[5] In Germany no tax on value or unrealized gain of value is levied at present (2016)

[6] ρ = 10% and σ=100% result in a period of 10 years for an increase by a factor of 2.7

[7] This is a most natural assumption for limited time periods. For low growth and *gt* << 1 the exponential

$$I(t) = I(0)e^{gt} \rightarrow$$
$$I_W(t) = I(0)e^{gt} - \rho C(0)e^{\rho\sigma t}$$

4.) Critical Situations and Piketty´s *Fundamental Rule*

A most critical situation would arise if all income would be absorbed by capital income

$$I_W(t) = 0 \rightarrow$$
$$t_1 = \frac{\ln(\rho \frac{C(0)}{I(0)})}{g - \rho\sigma}$$

A less, but also critical situation would arise if work income would stay flat with increasing total income

$$\frac{d\frac{I_W(t)}{I(0)}}{dt} = ge^{gt} - \rho^2\sigma\frac{C(0)}{I(0)}e^{\rho\sigma t}$$
$$\frac{d\frac{I_W(t)}{I(0)}}{dt} = 0 \rightarrow$$
$$t_2 = \frac{\ln(\rho^2\sigma\frac{C(0)}{I(0)})}{g - \rho\sigma}$$

For realistic values of ρ, σ and *C/I* the logarithm will be negative. The event to happen at a real time, the nominator must be negative: $g < \sigma\rho$. In other words: If capital interest should not absorb all Income (t_1) or all income increase (t_2), the growth rate must be above a certain level that depends on savings and interest rate: $g > \sigma\rho$.

This condition is a detailing of what happens if Piketty´s fundamental rule $r > g$ is true. What counts is not the general profit rate of the society r , but the profit rate of the *Have* segment ρ, multiplied by its capital saving rate σ .

For $\rho = 6\%$, $\sigma = 80\%$, $C(0)/I(0) = 5$; $g = 2\%$ work income would be zero after $t_1 \sim 150$ years. Such a situation would of course enforce political action long before it happens.

A more realistic event to happen is work income staying flat despite of growing total income. With the same parameters $t_2 \sim 40$ years. This is the time when work income stagnates while total income increases; later work income would decrease. Such a situation could be masked for some time by inflation, which feigns a virtual increase of work income.

When we set $t = 0$, we get the condition for no work income increase right from the start:

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growth pattern would be well approximated by a linear one. Here it is assumed, that all incomes develop at the same rate. In the simulation we also consider the case, where with decreasing total income, *have* can keep their income while *have-not* must take what is left. It will be seen that this does not create a marked difference.

$$t = 0 \rightarrow \ln(\frac{\sigma\rho^2}{g}\frac{C(0)}{I(0)}) = 0 \rightarrow \frac{\sigma\rho^2}{g}\frac{C(0)}{I(0)} = 1$$

$$g = \sigma\rho^2\frac{C(0)}{I(0)}$$

If growth is below this value, work income will never increase despite increasing total income. With the above parameters the critical growth rate of income would be 1.5%.

5.) Situations of Stability

What are conditions for absolute or relative stability?

(For clarity we now leave out the parenthesis *(t)*)

a.) Equal absolute growth of work and capital

$$I_W = I - I_C = I - \rho C$$

$$\frac{dI_W}{dt} = \frac{dI}{dt} - \rho\frac{dC}{dt} = g - .\rho\frac{dC}{dt}$$

$$\frac{dI_W}{dt} = \frac{dC}{dt}$$

$$\frac{dI_W}{dt} = g - \rho\frac{dI_W}{dt}$$

$$\rightarrow \frac{dI_W}{dt} = \frac{dC}{dt} = \frac{g}{1+\rho}$$

Without change in the distribution of capital the relation between work and capital income would be stable. Capital and work income would be growing at a rate slightly smaller than total income.

$$\frac{dC}{dt} = \sigma I_C$$

$$\sigma I_C(t) = \frac{g}{1+\rho} \rightarrow$$

$$\sigma = \frac{g}{1+\rho}\frac{1}{I_C}$$

With $I_C \gg 1$

$$\sigma \ll g$$

To achieve equal absolute growth the savings rate on capital would have to be very small, once sizable capital exists.

b) Equal relative growth

$$I_W = I - I_C = I - \rho C$$

$$\frac{dI_W}{dt} = Ig - .\rho\frac{dC}{dt}$$

$$\frac{dC}{dt} = \sigma\rho C$$

$$\frac{1}{I_W}\frac{dI_W}{dt} = \frac{1}{C}\frac{dC}{dt} \rightarrow$$
$$\frac{1}{I_W}(Ig - \sigma\rho^2 C) = \sigma\rho$$
$$\sigma = g\frac{I}{I_W}\frac{1}{\rho^2\frac{C}{I_W} + \rho}$$

With this saving rate capital and work income grow at the same rate

With $g = 3\%$; $I/I_w = 2$; $C/I_w = 8$, $\rho = 7\%$ σ should be 55%.

c) No further accumulation of capital

$$\frac{dC}{dt} = \sigma I_W + \sigma\rho C$$
$$\frac{dC}{dt} = 0 \rightarrow$$
$$1 \quad \sigma = 0$$
$$2 \quad \rho = 0 \text{ and no saving of work income}$$

There are 2 solutions

1 There is no (capital) saving; all income is joyously spent. This was indeed a typical behaviour of rich heirs in earlier centuries: *beware your own heritage for your successor, and enjoy its fruits in your lifetime*

2 Profit on capital is zero. This is the usury ban of the Old Testament, theoretically still valid in Mohammedanism.

d.) Exponential growth without flattening of work income

Capital is increasing more than work income, while work income keeps some increase for all times *($t_2 = \infty$)*

$$t_2 = \frac{\ln(\rho^2\sigma\frac{C(0)}{I(0)})}{g - \rho\sigma}$$
$$t_2 = +\infty \rightarrow$$
$$1 \quad g = \rho\sigma \text{ with } \ln(\rho^2\sigma\frac{C(0)}{I(0)}) \geq 0$$
$$g \leq \rho\sigma \text{ with } \ln(\rho^2\sigma\frac{C(0)}{I(0)}) = -\infty \rightarrow \rho^2\sigma\frac{C(0)}{I(0)} = 0$$
$$2 \quad \frac{C(0)}{I(0)} = 0$$
$$3 \quad \sigma = 0$$
$$4 \quad \rho = 0$$

Either $g = \sigma\rho$, or there is no initial Capital, or no saving, or no profit.

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6.) Simulation *Wealth Accumulation Part I*

The simulation is running in an Excel sheet. It is not using macros.

Have- Not is coined as A, *Have* as B.

The time period shown is 50 years. The parameters of the calculation are constant within this period

Starting condition at year zero:

Parameters and their range of variability

$0 \le C(0)/I(0) \le 5$	(in the property of B)
Total population	normalized to 1
$0.1 \le \varepsilon \le 0.000001$	B-fraction of the population
$90\% \le 1-\varepsilon \le 99.9999\%$	A-fraction of the population
$I(0)=1$	total income, normalized to 1
$I_{wA}(0)=1$	starting work income of A , normalized as 1
$10 \le \frac{I_{wB}(0)}{I_{wA}(0)} \le 1000$	multiple of B over A work income
$0 \le g \le 10\%$	exponential growth factor of total income
$0 \le s \le 100\%$	savings rate on capital income, optionally on work income too
$0 \le r \le 10\%$	capital profit rate

In the Excel sheet these parameters are easily and quickly changed by operating *sliders,* while the result is immediately visualized in graphs.

Two saving options can be chosen by activating *option fields*

- All work income of *A* and *B* is spent. The sole source of savings is capital income.
- Work income is saved by *B* at the same savings rate as capital income. This is of interest when there is little or no initial capital.

Normally it is assumed that work income of *A* and *B* change at the same rate. What if work income decreases? Here it may be more realistic to assume that *B* is able to defend its highest level at the cost of *A*. This behaviour can be simulated by activating a corresponding logic control switch (it will have a significant effect on *A* income, but not on capital).

Results of the simulations are shown in diagrams. While the Excel sheet is protected to avoid corruption, its graphs are not protected. They can be formatted and copied. One can delete lines to simplify graphs. One can let them reappear by the retro button. One can add lines by drawing the respective line of the calculation into a graph. You can generate a new graph in the non protected “working page” and copy lines of the calculation into it.

If you are interested in elaborating the algorithm, send a mail to get the protection code.

5.) Cases

To study the full range of cases covered by the algorithm, one should experiment with the simulation. Here some cases of basic interest are described, and are visualized by graphs generated with the simulation.

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A .) Generation of heirs and heiresses C(0)/I(0) = 3

C(0)/I(0)	3
ε	0,001
IA(0)	1
IB(0)	100
I(0)	1
g	4,00%
σ	100%
ρ	5,00%

Saving work income FALSCH

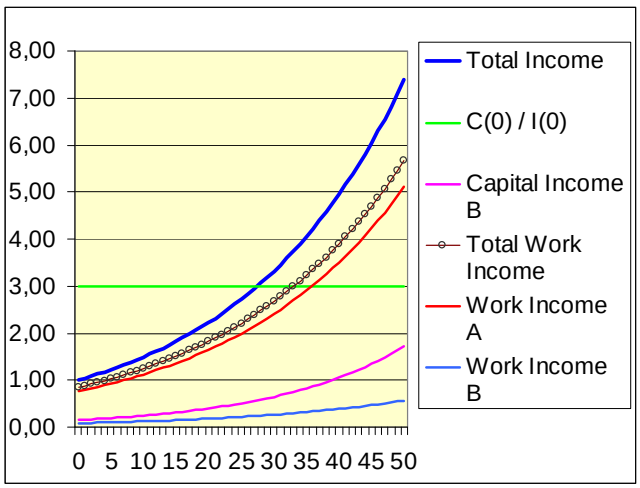
IW proportional

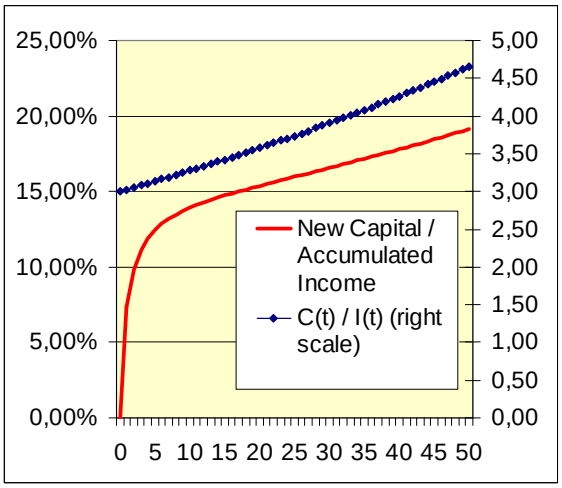
IW_B not reducing

Savings are of capital income only.

Work income of A and B develop proportionally.

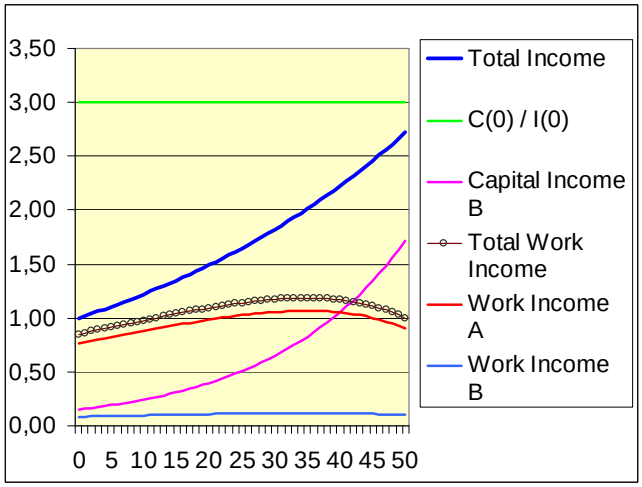
With these parameters the income situation at first glance looks inconspicuous. Total work income is not much below total income. The high income of a member of *B* (100 times *A*) is not highly visible as long as their number is sufficiently small ($\varepsilon * I(B)/I(A) << 1$). The *A* group gets by far most work income because of its high number.

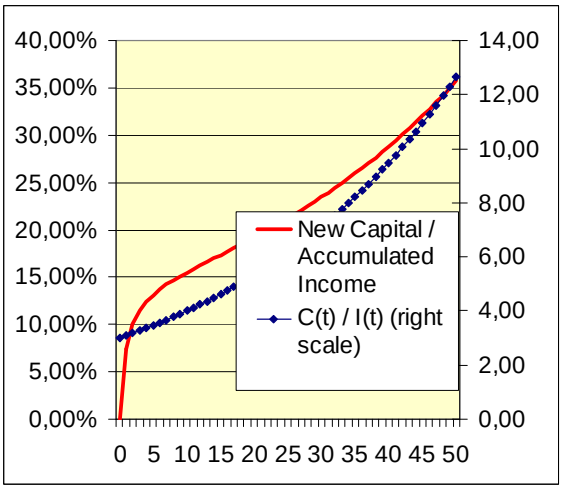




At the capital side *B* capital income is considerably higher than *B* work income from the start. As the right graph shows, the capital to income ratio rises from 3 to 4.5 in 50 years. Quite surprisingly, after 50 years close to 20% of all accumulated income of the period has been transferred to capital.

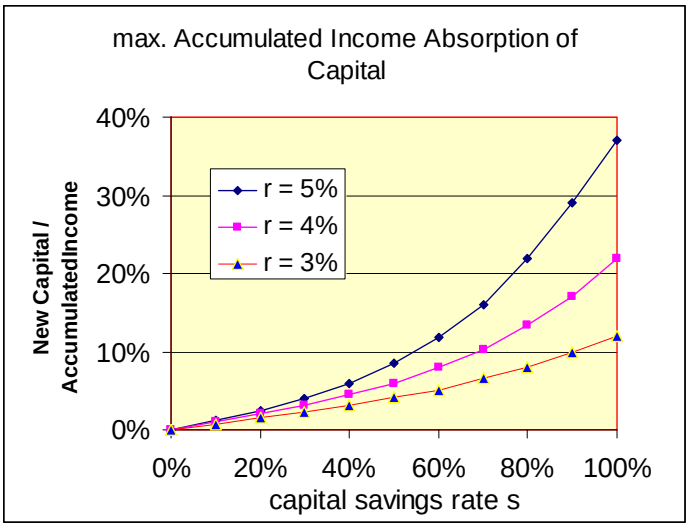
With a change oft growth from 4% to 2% the situation changes quite drastically. More and more of total income is needed for capital income, and work income stagnates after 30 years,





and would decline thereafter. The capital to income ratio rises from 3 to 12; accumulated capital absorbs 35% of accumulated income.

As derived above, capital increase by saved capital income is independent of income growth rate. This is the reason why the ratios explode with decreasing growth rate (one of Piketty's much disputed arguments).



The next graph shows, at $g = 2\%$, for different savings and interest rates how much of total income is absorbed over the period of 50 years by the interest on capital.

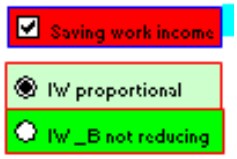
High profit rates assist a fast increase of the *new capital / accumulates income ratio*; yet the essential figure is the product of profit rate and savings rate $\rho\sigma$. If the savings rate of *B* is high, remarkable changes are happening even at low profit rates.

Piketty´s *Fundamental Rule r > g* (with r the profit rate of the total society) is not a necessary condition for wealth concentration

B.) Generation of Entrepreneurs

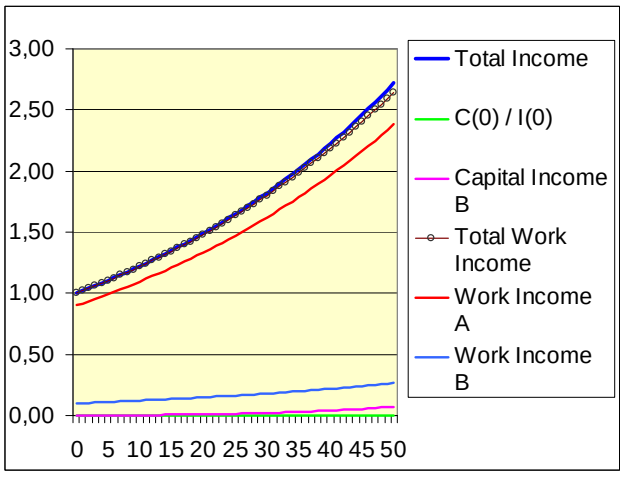
C(0)/I(0) = 0

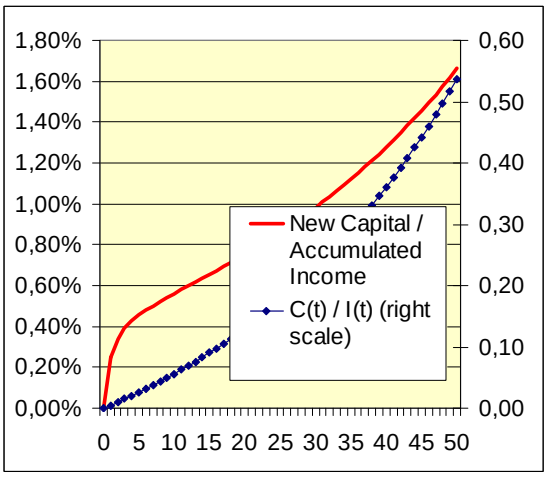
C(0)/I(0)	0
ε	0,001
IA(0)	1
IB(0)	100
I(0)	1
g	4,00%
σ	80%
ρ	5,00%



There is no starting capital (inheritance). Therefore there is no interest on starting capital. Now savings of B work income is important, which must be less than 100% to allow for living expenses.

If we assume work income to be as before, the following curves result:





Capital income of *B* rises from zero, but stays way below *B* work income. The right graph shows that after 50 years 1,6% of accumulated total income has turned into capital.

While there is some qualitative similarity to reality, the effect of capital accumulation is much too small. The reason for this is that equal growth of income is assumed for all. The work income of the typical successful, modern entrepreneur rises much faster, with average rates up

to 100% per year over longer periods[8]. So this two- segment model is not suited for simulation of this case. It will be treated in the many- segment model, where different growth rates are attributed to segments, while the whole society develops at a uniform low rate.

C.) Influence of B saving work income

C(0)/I(0)	3
ε	0,001
IA(0)	1
IB(0)	100
I(0)	1
g	2,00%
σ	80%
ρ	3,00%

Saving work income

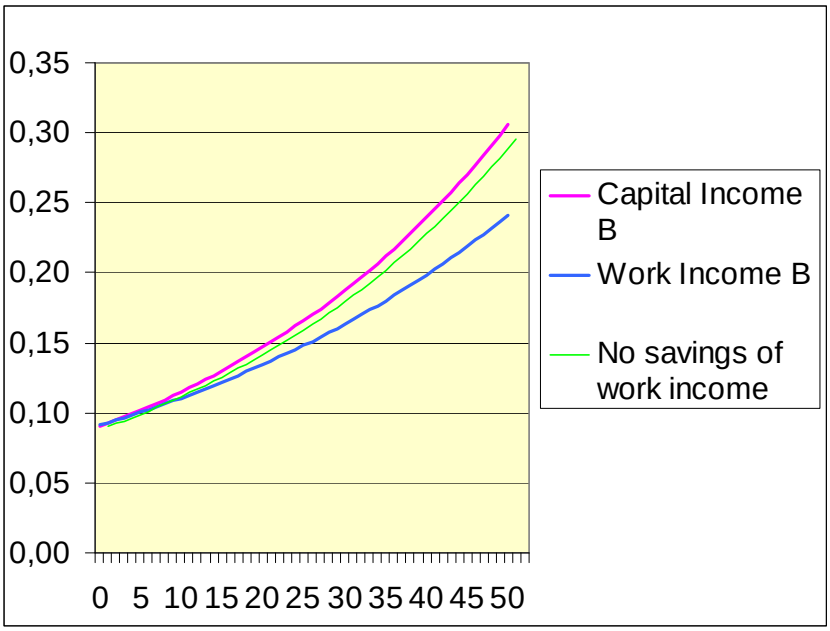
IW proportional

IW _B not reducing

This is the situation of heirs with high income. We ask if it makes a difference if they don't just save much of their capital income but also the same part of their work income.

Sources of B work income may be salary as executive, remuneration as member of boards and other organisations. Dividends and other income from capital are treated as capital income.

Parameters are such that capital income is equal to work income at the start.



The magenta curve shows capital income with savings of work income too, the green one with savings of capital income only. There is no really relevant difference, as the starting capital is so much higher than the savings of work income in one year (a factor of 30 with the chosen parameters)

$$\frac{\text{starting capital}}{\text{work income}} = \frac{C(0)/I(0)}{\varepsilon} * \frac{I_B(0)}{I(0)}$$

It makes no big difference in their capital growth if heirs in the definition of our model save work income or spent it. Their work income does not significantly contribute to their capital increase.

D.) Stagnation of work income with growing total income

As derived above analytically, with low growth of total income at some time the profit on capital will absorb all increases; work income will become flat and then decline. The formula for this critical time is

[8] If ones assumes a successful life period of 30 years between start at 100.000 income and arrival at 1 Billion accumulated income, constant growth per year of 38% would be sufficient, for a period of only 10 years one of 115% would be necessary - wonder of the exponential function!

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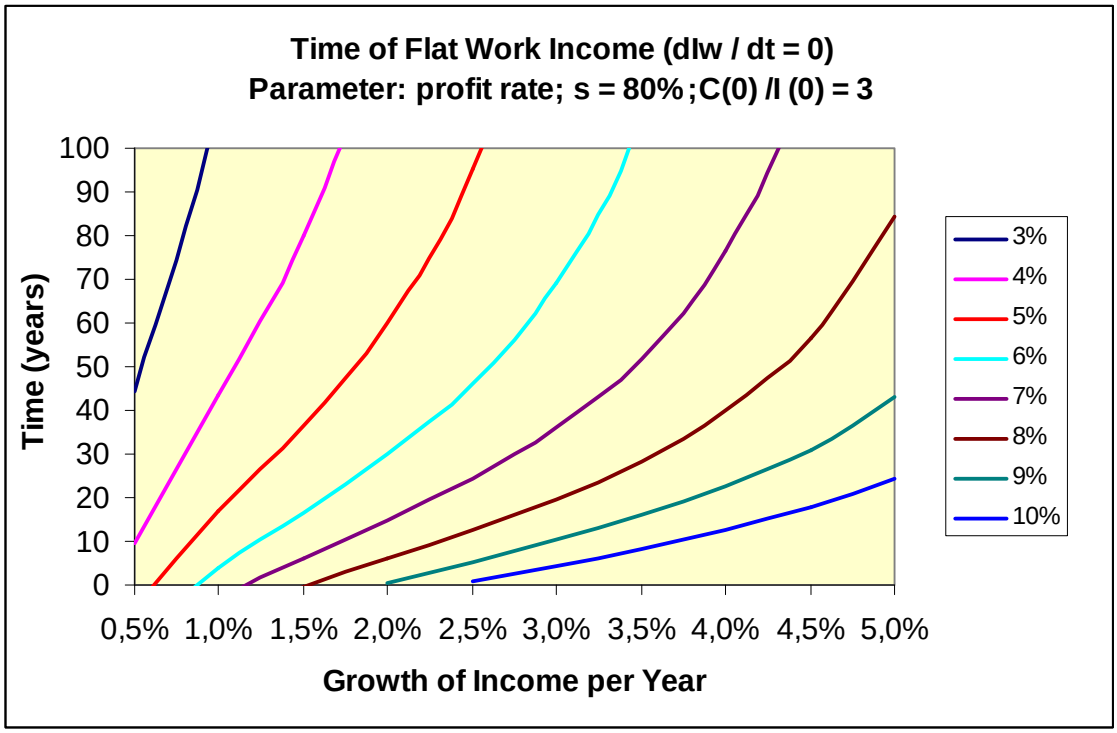
$$t = \frac{\ln(\frac{sr^2}{g}\frac{C(0)}{I(0)})}{g - sr}$$

The following two graphs visualize results of it.

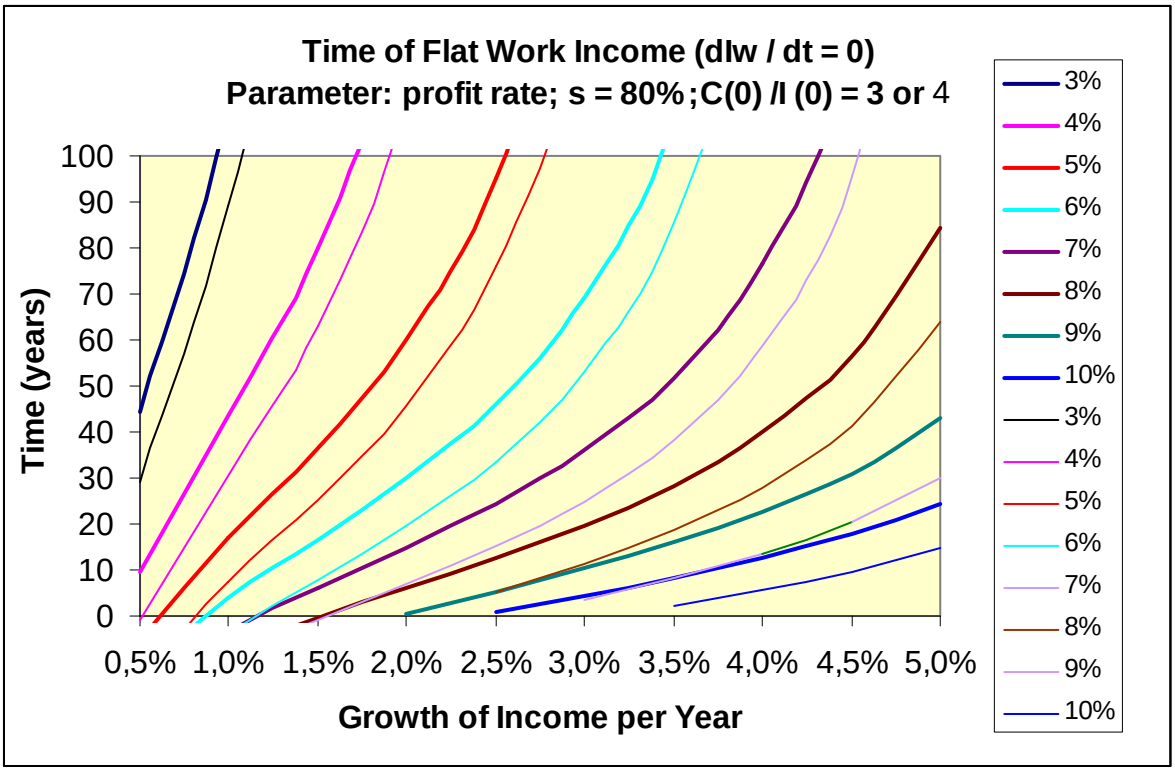
In the plot the growth parameter $0.5\% \le g \le 5\%$ runs along the abscissa; the profit rate $3\% \le r \le 10\%$ is parameter of the different curves. The savings rate $s = 80\%$ on capital profit is constant.

In the first graph the initial Capital to Income ratio is $C(0) / I(0) = 3$.



At the low (real) growth rates and typical capital to income ratios of modern capitalistic societies it takes several decenniums for work income to flatten, even at moderate profit rates of capital.

The last graph illustrates what happens if the initial capital to income rate is a bit higher (4 instead of 3). The added thin lines illustrate the change against the situation with $C(9) / I(0) = 3$. It is quite drastic at low growth and characterizes a later period in an otherwise unchanged process of wealth accumulation.



5.) Critical Discussion

Applicability of the 2 segment model

As discussed above, the model is useful for observing the situation of heirs and heiresses, and the one of entrepreneurs after they have accumulated a comparable capital by saving of very high work income. The model is not useful for describing the entrepreneurial run-up phase – this needs a many-segment model with very high work income growth in the entrepreneurial segment (Part II).

It was demonstrated that under these restrictions saving of work income plays a limited role in capital accumulation. Naturally work income of B as a segment of the society will be small compared to that of A. In context with capital accumulation it is not significant how high the work income of B individuals is or how many are dividing a certain capital.

How realistic are the assumptions concerning the parameter range?

In this model the initial capital is the one used to create more capital. Hence the capital to income ratio is smaller than the one reported in official statistics

Besides the initial capital to income ratio the critical parameters are growth rate g of income, savings rate σ of B and profitability of B capital ρ. As derived above, capital accumulation is governed by the product $\rho\sigma$, it is independent of the growth rate g which enters critically into the division of total income into work and capital income and hence is politically most significant.

Growth rate *g*

In today's western capitalistic countries net growth of total income – which we roughly set equal to GNP (BIP) – is quite low, at most a few percent. For Germany recently it is around 1.6%.

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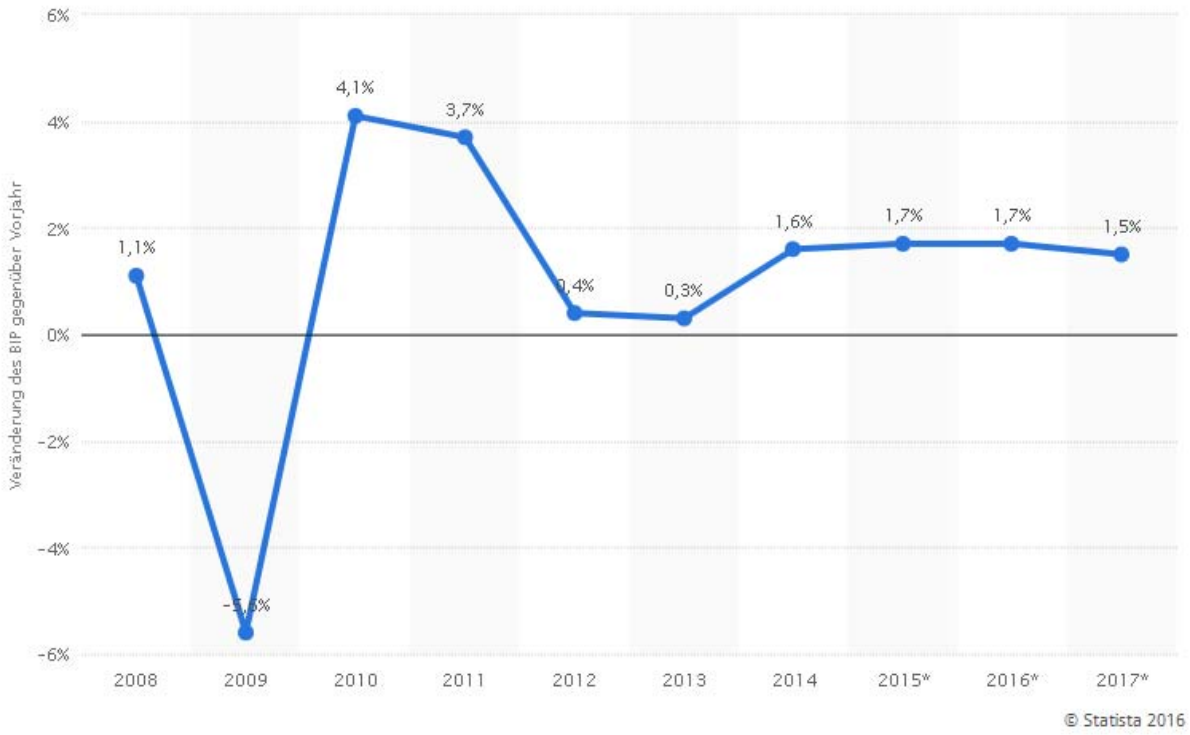
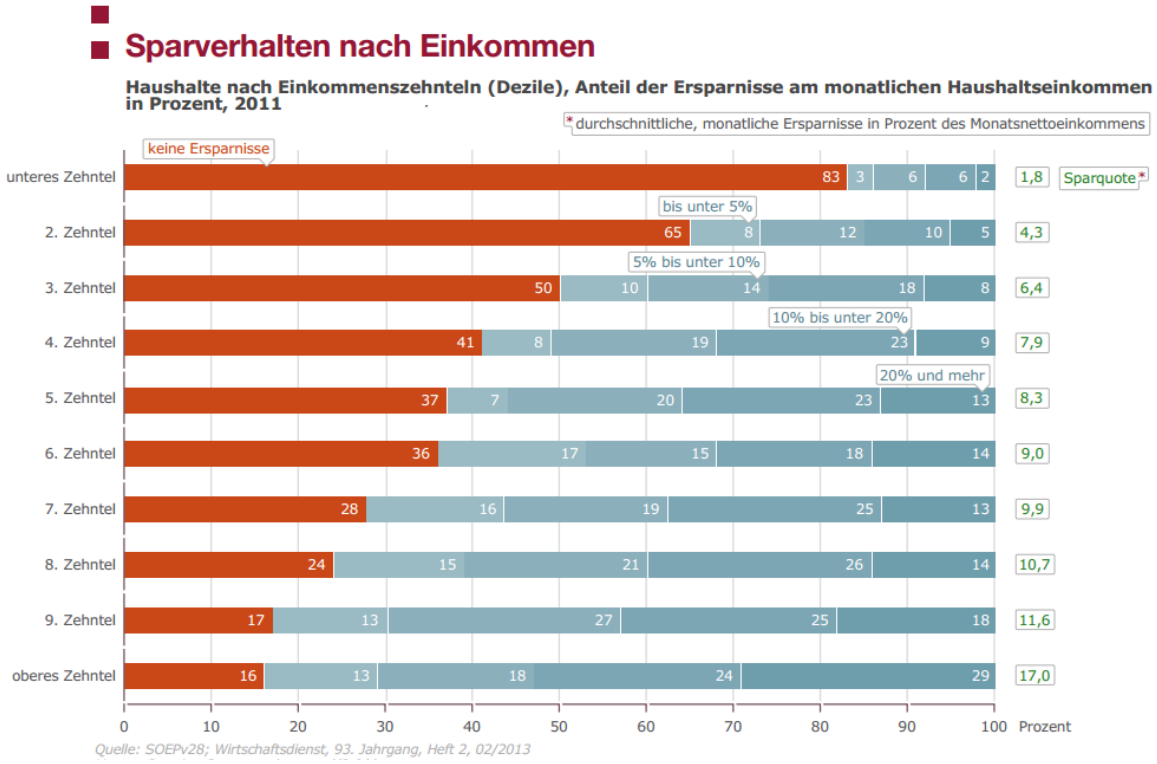


Savings rate σ

The average saving rate in Germany has been close to 10% for many years, with a spread between different income deciles between 1.8 and 17%.
https://www.bpb.de/system/files/dokument.../08%20Sparverhalten.pdf



Such a moderate value would not justify the high rates that we use in the analysis. But it is misleading for two reasons: it does not include the increase in the value of capital investments, and it does not differentiate inside of the upper decile, where capital accumulation is very unequal.

So we better make an "educated guess", starting with the following parameters:

C(0)/I(0)	2
ε	0,001
IA(0)	1
IB(0)	30
I(0)	1
g	1,50%
s	80%
r	5,00%

and the following facts:
Population of Germany 81 Million
BIP 3.600.000 Million €
Setting BIP ~ Income I(0) results in
I_W(A) per capita ~ 40 T €
I_W(B) per capita ~ 1.2 Mio €
C(0) per capita (B) ~ 90 Mio €
Capital income per capita (B) ~4.5 Mio €

It is insignificant that these numbers may be uncertain by a small factor, because of a possible difference between *gross* and *net* and because of the common inclusion of capital goods in capital that we treat as consumables. It is obvious anyway that A will not accumulate substantial capital by saving part of work income. It is equally obvious that B can live a most comfortable life by spending work income and saving 100% of capital income. So calculating with high savings rates of B is justified.

Profit rate r

How about the profit rate ρ? For A the profit rate of achievable investments presently ranges between 0 for bank accounts and 3 to 4% (minus 25% tax[9]) as dividends of traditional DAX-shares.

This is in striking contrast to the 20%- goal that Chairman Josef Ackermann of Deutsche Bank set some years ago for the "Eigenkapitalrendite" (~ profit rate of the bank owners), *to catch up to US banks*.

No data are documented on the profit rate that B can achieve on average. Yet sufficient material has been published in the newspapers to estimate it for the top class of German Billionaires (in German: *Milliardäre,* 123 at present*)*.

In the following table a few prominent ones have been selected for which data for longer periods are available. Three of these are heirs (Klatten, Quandt, Kühne), the other are entrepreneurs. This demonstrates the interest to also analyze the conditions of entrepreneurial capital accumulation.

[9] Abgeltungssteuer

Source https://de.wikipedia.org/wiki/Chronologie_der_reichsten_Deutschen

Wealth in Billion € of some of the richest Germans

	Jahr 2004	2008	**average yearly growth**
Karl Albrecht	15,6	27	14%
	Jahr 2004	2011	
Susanne Klatten	7,8	14,6	9%
	Jahr 2004	2007	
Andreas und Thomas Strüngmann	2,8	8	35%
	Jahr 2004	2015	
Hasso Plattner	7	9,1	2%
	Jahr 2004	2015	
Johanna Quandt	4	13,9	11%
	Jahr 2007	2015	
Klaus und Michael Kühne	5,9	11,9	9%
	Jahr 2004	2007	
Reinhold Würth	5,05	9	19%

The differential average yearly growth rate has been calculated for the time period stated. Some may be distorted by non-periodic events; yet a mean value of about 10% seems typical. The values are net.

Thus it is justified to calculate with $s \leq 100\%$ and $r \leq 10\%$ in the model.

Ways out of the *capitalistic trap* (flattening A work income)

In the last chapter *Regulating Capital in the Twenty-First Century* (page 469 onward) Piketty discusses possible ways to stop or at least decelerate an assumed increasing inequality of capital. As ideal he describes a global tax on capital. He calls this a utopian goal, which yet could be a guideline for a stepwise change in interested groups of nations, such as the EU. As suitable ranges he quotes 0% for fortunes up to 1 Million, 1% for 1 to 5 Million, 2 % for more than 5 Million and finally 5 to 10% for more than 1 Billion. In addition he pleads for a progressive income tax.

The argument for such high taxes would not be increasing the total state income substantially or creating sufficient means for subsidizing underprivileged citizens – for these purposes the number of involved persons is too small. The purpose would be to avoid politically dangerous developments, staying closer to the ideal of a democratic egalitarian or meritocratic society.

Our simple model gives indications of the range of measures necessary to pursue such a goal.

B capital increases at a rate of $\sigma\rho$, A income at less than g. In order to prevent a further discrepancy between the increase of capital and of income, capital tax rate would have to be of the order

$$\sigma\rho - g$$

To arrive at realistic numbers, one would have to analyze the savings and the profit possibilities at different capital levels. If we just take the data quoted for German Billionaires ($r \sim 10\%$, $s \sim 100\%$) and $g \sim 8\%$ a capital tax rate of 8% would achieve the goal for that group. This is in line with Piketty´ value. Big capital would grow, but no faster than incomes.

Since today there is practically no capital tax in Germany, any measure in this direction would naturally meet massive resistance by those involved. Higher taxes on income might be less sensitive, as (except of entrepreneurs) B income does not significantly contribute to B capital growth.

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Striving for an e*qual chance* society presupposes a highly progressive tax on heritage; Piketty states examples of extreme rates in different countries and periods. In Germany heritage taxes are very moderate at present.

These considerations concern the question which kind of society citizens want. Yet this is a political question beyond a *sine ira et studio* analysis.

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